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Proposed operator description of four dimensional Euclidean quantum gravity

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Context: Quantum gravity

- “4D Euclidean quantum gravity” (EQG)
 - Gibbons, Hawking: “Euclidean Quantum Gravity” [1]
 - ed., (1993), Singapore: World Scientific
 - Field theory / second quantization
 - Expectation: Operator description of 4D EQG unlikely
 - Suggestion for realization [2]: anti-de Sitter (AdS) metric
- Research problem of this talk: Find operator (first quantization) description

Context: Standard Model

Operator description
(1st quantization)

Field theory
(2nd quantization)

Electromagnetism



Electromagnetism

Strong force
Weak force



Proposed: Standard Model + EQG

Operator description
(1st quantization)

Field theory
(2nd quantization)

Electromagnetism



Electromagnetism



Strong force
Weak force



Proposed here:
4D EQG

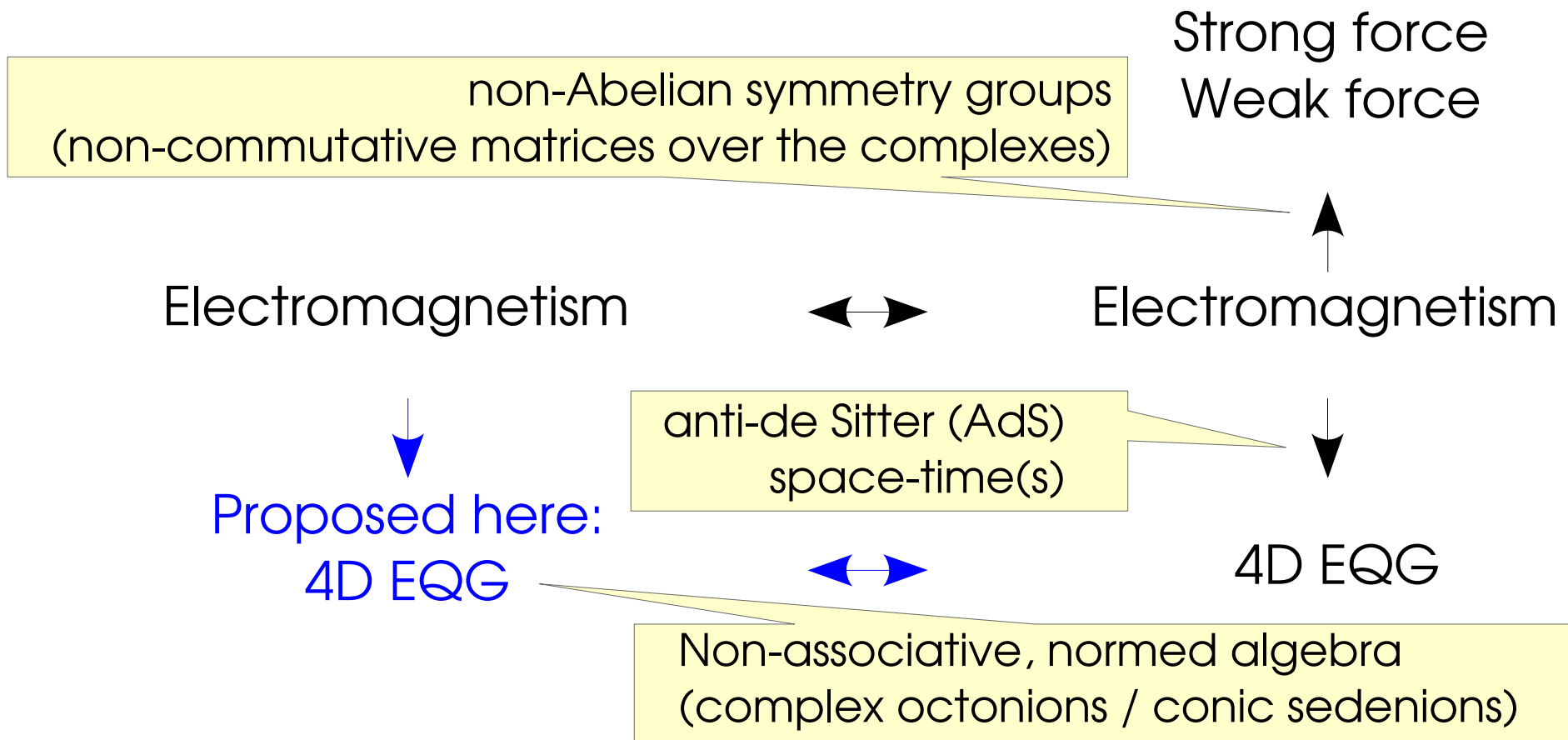


4D EQG

Computation concepts

Operator description
(1st quantization)

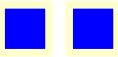


Field theory
(2nd quantization)



Motivation

- Why normed algebra?
 - Use metric to define relativity: Different frames of reference are equivalent when energy-momentum vector norm is invariant.
- Why non-associative algebra?
 - Consistent with known physics: Octonionic expression of QED (Gogberashvili [3,4], Köpflinger [8])
 - Octonionic Proca-Maxwell equations [9]
 - “New” (non-matrix) symmetry transformations possible (Köpflinger [5-8])

Cayley-Dickson constructs ...

(real)	1 i_1 i_2 i_3 i_4 i_5 i_6 i_7	
Complex		
Quaternion		
Octonion		

... and “complexified” algebras

(real)	$1 \ i_1 \ i_2 \ i_3 \ i_4 \ i_5 \ i_6 \ i_7 \ i_0 \ \varepsilon_1 \ \varepsilon_2 \ \varepsilon_3 \ \varepsilon_4 \ \varepsilon_5 \ \varepsilon_6 \ \varepsilon_7$	(complex)
<i>Complex</i>		Tessarine (or conic quat., bicomplex num)
<i>Quaternion</i>		Biquaternion (or compl. quat., conic octonion)
<i>Octonion</i>		Complex octonion (or conic sedenion)

Spin 1/2 particle w/o field









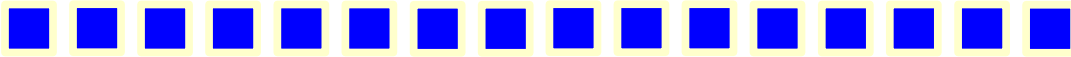
	1	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_0	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	
Gogberashvili [3]	■	■	■	■									■	■	■	■	Dirac eqn
Köplinger [5]	■	■	■	■									■	■	■	■	Dirac eqn

$$\Psi_{\text{hyp8}} := \left(\psi_0^r, \psi_0^i, \psi_1^r, \psi_1^i, \quad \psi_2^r, -\psi_2^i, -\psi_3^r, -\psi_3^i \right)$$

$$\nabla_{\text{hyp8}} := \left(-m, \partial_0, 0, 0, \quad 0, -\partial_3, \partial_2, -\partial_1 \right)$$

$$\nabla_{\text{hyp8}} \Psi_{\text{hyp8}} = 0$$

Spin 1/2 particle with field

	1 i ₁ i ₂ i ₃ i ₄ i ₅ i ₆ i ₇ i ₀ ε ₁ ε ₂ ε ₃ ε ₄ ε ₅ ε ₆ ε ₇	
Gogberashvili [3]	 	Dirac eqn
Gogberashvili [4]	 	EM
Köplinger [5]	 	Dirac eqn
Köplinger [6]	<p>“New” (non-matrix) symmetry trans.:</p> 	“hvy” Dirac
Köplinger [7]	<p>Define consistent “relativity”:</p> 	GRT equiv
Köplinger [8]	<p>Add EM field consistent with QED:</p> 	EM & GR

Consistency (non-quantum)

- How would we observe a complex metric that is both Euclidean and Minkowskian?
 - Large body, non-quantum effects don't mix forces; we can first look at Euclidean and Minkowski metrics separately, and then project the effects from Euclidean geometry onto Minkowskian geometry.
 - Supporting argument: Our daily experience is dominated by Electromagnetism and Minkowski space-time, other forces may appear as a modification thereof.

Model: NatAliE equations

- “Naturally aligned elementary equations”
 - Treat gravity on global 4D Euclidean space-time, separately from all other forces.
 - For this space-time only, develop relativity in direct correspondence to Special Relativity (SRT):
 - Unaccelerated frames of reference are equivalent.
 - “Heavy” Lorentz transformation conserves 4D Euclidean length element and energy-momentum.
 - As little changes as possible compared to SRT.
 - In the SRT equation of motion, modify the gravitational field geometry accordingly.


Result (non-quantum)

- NatAliE equations become linearized field equations from General Relativity (GRT) [7]
 - Since we did not look at field self-interaction, this is the desired result (for consistency).
 - Known “bootstrap” method yields covariant GRT.
 - Conclusion: NatAliE is consistent with experimental results for large body, non-quantum physics [7].
- Now: generalize for quantum physics

Model for quantum gravity

- Perform operator transition [8]:
 - Conic sedenions (complex octonions) as enveloping algebra that contains both Euclidean and Minkowskian geometry.
 - Add EM field consistent with QED.
 - Introduce “mixing angle” to transition between electromagnetism and gravity.
- Relativity warranted by invariant modulus (norm) of the energy-momentum vector.

Spin 1/2 particle with field

	1 i_1 i_2 i_3 i_4 i_5 i_6 i_7 i_0 ϵ_1 ϵ_2 ϵ_3 ϵ_4 ϵ_5 ϵ_6 ϵ_7	
Köplinger [8]		EM & GR

$$\nabla_{Q1} := (0, \partial_0, 0, 0, 0, 0, 0, 0, 0, 0, eA_0, 0, 0, 0, 0, 0, 0, 0)$$

$$\nabla_{Q2} := (0, 0, 0, 0, 0, \partial_3, -\partial_2, \partial_1, 0, 0, 0, 0, 0, 0, eA_3, -eA_2, eA_1)$$

$$\nabla := \nabla_{Q1} + \exp(i_0\alpha) \nabla_{Q2}$$

$$\Psi_{Q1} := (\psi_0^r, \psi_0^i, \psi_1^r, \psi_1^i, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$$

$$\Psi_{Q2} := (0, 0, 0, 0, -\psi_2^r, \psi_2^i, \psi_3^r, \psi_3^i, 0, 0, 0, 0, 0, 0, 0, 0, 0)$$

$$\Psi := \Psi_{Q1} + \exp(i_0\alpha) \Psi_{Q2}$$

$$(\nabla - m) \Psi = 0$$

Predictions

- Challenge: Quantum gravity's weakness
 - Conic sedenion formulation reduces to tensor algebra in excellent approximation for current experiments [8]. This is a challenge in validating any theory on quantum gravity.
- Initial findings
 - Backscattering is increased for elastic gravitational spin $\frac{1}{2}$ scattering
 - Scattering cross section does not vanish for $E \rightarrow \infty$

Outlook

- To examine:
 - Measurable effects that distinguish this model from other models on quantum gravity?
 - Vacuum effects?
 - Extendability of Gogberashvili's approach?
 - Effects at Big Bang energy levels?
 - Massive exchange particle(s)?
 - Other forces?

Acknowledgments

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for helping me understand and utilize algebraic concepts, and for vital and fruitful discussions.

Appendix: Conic sedenions

Quick notes about conic sedenions (complex octonions) [10,11]

Modulus (norm): $|z| = |a + \sum b_n i_n + \sum c_n \varepsilon_n + d| := \sqrt[4]{(a^2 + b_n^2 - c_n^2 - d^2)^2 + 4(ad - b_n c_n)^2}$

Multiplication table:

X	1	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_0	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7
1	1	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_0	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7
i_1	i_1	-1	i_3	$-i_2$	i_5	$-i_4$	$-i_7$	i_6	$-i_0$	i_0	ε_3	$-\varepsilon_2$	ε_5	$-\varepsilon_4$	$-\varepsilon_7$	ε_6
i_2	i_2	$-i_3$	-1	i_1	i_6	i_7	$-i_4$	$-i_5$	$-\varepsilon_2$	$-\varepsilon_3$	i_0	ε_1	ε_6	ε_7	$-\varepsilon_4$	$-\varepsilon_5$
i_3	i_3	i_2	$-i_1$	-1	i_7	$-i_6$	i_5	$-i_4$	$-\varepsilon_3$	ε_2	$-\varepsilon_1$	i_0	ε_7	$-\varepsilon_6$	ε_5	$-\varepsilon_4$
i_4	i_4	$-i_5$	$-i_6$	$-i_7$	-1	i_1	i_2	i_3	$-\varepsilon_4$	$-\varepsilon_5$	$-\varepsilon_6$	$-\varepsilon_7$	i_0	ε_1	ε_2	ε_3
i_5	i_5	i_4	$-i_7$	i_6	$-i_1$	-1	$-i_3$	i_2	$-\varepsilon_5$	ε_4	$-\varepsilon_7$	ε_6	$-\varepsilon_1$	i_0	$-\varepsilon_3$	ε_2
i_6	i_6	i_7	i_4	$-i_5$	$-i_2$	i_3	-1	$-i_1$	$-\varepsilon_6$	ε_7	ε_4	$-\varepsilon_5$	$-\varepsilon_2$	ε_3	i_0	$-\varepsilon_1$
i_7	i_7	$-i_6$	i_5	i_4	$-i_3$	$-i_2$	i_1	-1	$-\varepsilon_7$	$-\varepsilon_6$	ε_5	ε_4	$-\varepsilon_3$	$-\varepsilon_2$	ε_1	i_0
i_0	i_0	$-\varepsilon_1$	$-\varepsilon_2$	$-\varepsilon_3$	$-\varepsilon_4$	$-\varepsilon_5$	$-\varepsilon_6$	$-\varepsilon_7$	-1	i_1	i_2	i_3	i_4	i_5	i_6	i_7
ε_1	ε_1	i_0	ε_3	$-\varepsilon_2$	ε_5	$-\varepsilon_4$	$-\varepsilon_7$	ε_6	i_1	1	$-i_3$	i_2	$-i_5$	i_4	i_7	$-i_6$
ε_2	ε_2	$-\varepsilon_3$	i_0	ε_1	ε_6	ε_7	$-\varepsilon_4$	$-\varepsilon_5$	i_2	i_3	1	$-i_1$	$-i_6$	$-i_7$	i_4	i_5
ε_3	ε_3	ε_2	$-\varepsilon_1$	i_0	ε_7	$-\varepsilon_6$	ε_5	$-\varepsilon_4$	i_3	$-i_2$	i_1	1	$-i_7$	i_6	$-i_5$	i_4
ε_4	ε_4	$-\varepsilon_5$	$-\varepsilon_6$	$-\varepsilon_7$	i_0	ε_1	ε_2	ε_3	i_4	i_5	i_6	i_7	1	$-i_1$	$-i_2$	$-i_3$
ε_5	ε_5	ε_4	$-\varepsilon_7$	ε_6	$-\varepsilon_1$	i_0	$-\varepsilon_3$	ε_2	i_5	$-i_4$	i_7	$-i_6$	i_1	1	i_3	$-i_2$
ε_6	ε_6	ε_7	ε_4	$-\varepsilon_5$	$-\varepsilon_2$	ε_3	i_0	$-\varepsilon_1$	i_6	$-i_7$	$-i_4$	i_5	i_2	$-i_3$	1	i_1
ε_7	ε_7	$-\varepsilon_6$	ε_5	ε_4	$-\varepsilon_3$	$-\varepsilon_2$	ε_1	i_0	i_7	i_6	$-i_5$	$-i_4$	i_3	i_2	$-i_1$	1

Other relations:

$$e^{\varepsilon_n \alpha} = \cosh \alpha + \varepsilon_n (\sinh \alpha)$$

$$\ln \varepsilon_n = \frac{\pi}{2} (i_0 - i_n)$$

$$\varepsilon_n^\alpha = \frac{1}{2} [(1 - \varepsilon_n) + (1 + \varepsilon_n) e^{-\pi i_n \alpha}]$$

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