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Proposed operator description
of
four dimensional Euclidean quantum gravity

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Context: Quantum gravity

- "4D Euclidean quantum gravity" (EQG)
  - Gibbons, Hawking: "Euclidean Quantum Gravity" [1]
    - ed., (1993), Singapore: World Scientific
  - Field theory / second quantization
  - Expectation: Operator description of 4D EQG unlikely
  - Suggestion for realization [2]: anti-de Sitter (AdS) metric
- Research problem of this talk: Find operator (first quantization) description

Context: Standard Model

Operator description (1st quantization)		Field theory (2nd quantization)
		Strong force
		Weak force
		↑
Electromagnetism	↔	Electromagnetism

Proposed: Standard Model + EQG

Operator description (1^{st} quantization)		Field theory (2^{nd} quantization)
		Strong force Weak force
		↑
Electromagnetism	↔	Electromagnetism
↓		↓
Proposed here: 4D EQG	↔	4D EQG

Computation concepts

Operator description
(1st quantization)

Field theory
(2nd quantization)

Strong force
Weak force

non-Abelian symmetry groups
(non-commutative matrices over the complexes)

↑

Electromagnetism ↔ Electromagnetism

↓ ↓

anti-de Sitter (AdS)
space-time(s)

Proposed here:
4D EQG ↔ 4D EQG

Non-associative, normed algebra
(complex octonions / conic sedenions)

Motivation

- Why normed algebra?
  - Use metric to define relativity: Different frames of reference are equivalent when energy-momentum vector norm is invariant.
- Why non-associative algebra?
  - Consistent with known physics: Octonionic expression of QED (Gogberashvili [3,4], Köplinger [8])
  - Octonionic Proca-Maxwell equations [9]
  - "New" (non-matrix) symmetry transformations possible (Köplinger [5-8])

Cayley-Dickson constructs ...

(real)	1 i_1 i_2 i_3 i_4 i_5 i_6 i_7	
Complex	■ ■	
Quaternion	■ ■ ■ ■	
Octonion	■ ■ ■ ■ ■ ■ ■ ■	

... and "complexified" algebras

(real)	1	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_0	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	(complex)
Complex	■	■							■	■							Tessarine (or conic quat., bicomplex num)
Quaternion	■	■	■	■					■	■	■	■					Biquaternion (or compl. quat., conic octonion)
Octonion	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	Complex octonion (or conic sedenion)

Spin ½ particle w/o field

	1	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_0	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	
Gogberashvili [3]	■	■	■	■									■	■	■	■	Dirac eqn
Köplinger [5]	■	■	■	■									■	■	■	■	Dirac eqn

$$\Psi_{\text{hyp8}} := \left(\psi_0^{\text{r}}, \psi_0^{\text{i}}, \psi_1^{\text{r}}, \psi_1^{\text{i}}, \quad \psi_2^{\text{r}}, -\psi_2^{\text{i}}, -\psi_3^{\text{r}}, -\psi_3^{\text{i}}\right)$$

$$\nabla_{\text{hyp8}} := (-m, \partial_0, 0, 0, \quad 0, -\partial_3, \partial_2, -\partial_1)$$

$$\nabla_{\text{hyp8}} \Psi_{\text{hyp8}} = 0$$

Spin ½ particle with field

	1	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_0	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	
Gogberashvili [3]	■	■	■	■									■	■	■	■	Dirac eqn
Gogberashvili [4]	■	■	■	■									■	■	■	■	EM
Köplinger [5]	■	■	■	■									■	■	■	■	Dirac eqn
	“New“ (non-matrix) symmetry trans.:																
Köplinger [6]	■	■	■	■	■	■	■	■									“hvy“Dirac
	Define consistent “relativity“:																
Köplinger [7]	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	GRT equiv
	Add EM field consistent with QED:																
Köplinger [8]	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	EM & GR

Consistency (non-quantum)

- How would we observe a complex metric that is both Euclidean and Minkowskian?
  - Large body, non-quantum effects don't mix forces; we can first look at Euclidean and Minkowski metrics separately, and then project the effects from Euclidean geometry onto Minkowskian geometry.
  - Supporting argument: Our daily experience is dominated by Electromagnetism and Minkowski space-time, other forces may appear as a modification thereof.

Model: NatAliE equations

- “Naturally aligned elementary equations”
  - Treat gravity on global 4D Euclidean space-time, separately from all other forces.
  - For this space-time only, develop relativity in direct correspondence to Special Relativity (SRT):
    - Unaccelerated frames of reference are equivalent.
    - “Heavy” Lorentz transformation conserves 4D Euclidean length element and energy-momentum.
    - As little changes as possible compared to SRT.
  - In the SRT equation of motion, modify the gravitational field geometry accordingly.

Result (non-quantum)

- NatAliE equations become linearized field equations from General Relativity (GRT) [7]
  - Since we did not look at field self-interaction, this is the desired result (for consistency).
  - Known "bootstrap" method yields covariant GRT.
  - Conclusion: NatAliE is consistent with experimental results for large body, non-quantum physics [7].
- Now: generalize for quantum physics

Model for quantum gravity

- Perform operator transition [8]:
  - Conic sedenions (complex octonions) as enveloping algebra that contains both Euclidean and Minkowskian geometry.
  - Add EM field consistent with QED.
  - Introduce "mixing angle" to transition between electromagnetism and gravity.

- Relativity warranted by invariant modulus (norm) of the energy-momentum vector.

Spin ½ particle with field

	1 i$_1$ i$_2$ i$_3$ i$_4$ i$_5$ i$_6$ i$_7$ i$_0$ ε$_1$ ε$_2$ ε$_3$ ε$_4$ ε$_5$ ε$_6$ ε$_7$	
Köplinger [8]	■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■	EM & GR

$$\nabla_{Q1} := (0, \partial_0, 0, 0, 0, 0, 0, 0, \quad 0, eA_0, 0, 0, 0, 0, 0, 0)$$
$$\nabla_{Q2} := (0, 0, 0, 0, 0, \partial_3, -\partial_2, \partial_1, \quad 0, 0, 0, 0, 0, eA_3, -eA_2, eA_1)$$
$$\nabla := \nabla_{Q1} + \exp(i_0 \alpha)\, \nabla_{Q2}$$

$$\Psi_{Q1} := \left(\psi_0^{\mathrm{r}}, \psi_0^{\mathrm{i}}, \psi_1^{\mathrm{r}}, \psi_1^{\mathrm{i}}, 0, 0, 0, 0, \quad 0, 0, 0, 0, 0, 0, 0, 0\right)$$
$$\Psi_{Q2} := \left(0, 0, 0, 0, -\psi_2^{\mathrm{r}}, \psi_2^{\mathrm{i}}, \psi_3^{\mathrm{r}}, \psi_3^{\mathrm{i}}, \quad 0, 0, 0, 0, 0, 0, 0, 0\right)$$
$$\Psi := \Psi_{Q1} + \exp(i_0 \alpha)\, \Psi_{Q2}$$

$$\left(\nabla - m\right) \Psi = 0$$

Predictions

- Challenge: Quantum gravity's weakness
  - Conic sedenion formulation reduces to tensor algebra in excellent approximation for current experiments [8]. This is a challenge in validating any theory on quantum gravity.
- Initial findings
  - Backscattering is increased for elastic gravitational spin ½ scattering
  - Scattering cross section does not vanish for E --> ∞

Outlook

- To examine:
  - Measurable effects that distinguish this model from other models on quantum gravity?
  - Vacuum effects?
  - Extendability of Gogberashvili's approach?
  - Effects at Big Bang energy levels?
  - Massive exchange particle(s)?
  - Other forces?

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Appendix: Conic sedenions

Quick notes about conic sedenions (complex octonions) [10,11]

Modulus (norm): $|z| = |a + \sum b_n i_n + \sum c_n \varepsilon_n + d| := \sqrt[4]{(a^2 + b_n^2 - c_n^2 - d^2)^2 + 4(ad - b_n c_n)^2}$

Multiplication table:

x	1	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_0	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7
1	1	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_0	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7
i_1	i_1	-1	i_3	$-i_2$	i_5	$-i_4$	$-i_7$	i_6	$-\varepsilon_1$	i_0	ε_3	$-\varepsilon_2$	ε_5	$-\varepsilon_4$	$-\varepsilon_7$	ε_6
i_2	i_2	$-i_3$	-1	i_1	i_6	i_7	$-i_4$	$-i_5$	$-\varepsilon_2$	$-\varepsilon_3$	i_0	ε_1	ε_6	ε_7	$-\varepsilon_4$	$-\varepsilon_5$
i_3	i_3	i_2	$-i_1$	-1	i_7	$-i_6$	i_5	$-i_4$	$-\varepsilon_3$	ε_2	$-\varepsilon_1$	i_0	ε_7	$-\varepsilon_6$	ε_5	$-\varepsilon_4$
i_4	i_4	$-i_5$	$-i_6$	$-i_7$	-1	i_1	i_2	i_3	$-\varepsilon_4$	$-\varepsilon_5$	$-\varepsilon_6$	$-\varepsilon_7$	i_0	ε_1	ε_2	ε_3
i_5	i_5	i_4	$-i_7$	i_6	$-i_1$	-1	$-i_3$	i_2	$-\varepsilon_5$	ε_4	$-\varepsilon_7$	ε_6	$-\varepsilon_1$	i_0	$-\varepsilon_3$	ε_2
i_6	i_6	i_7	i_4	$-i_5$	$-i_2$	i_3	-1	$-i_1$	$-\varepsilon_6$	ε_7	ε_4	$-\varepsilon_5$	$-\varepsilon_2$	ε_3	i_0	$-\varepsilon_1$
i_7	i_7	$-i_6$	i_5	i_4	$-i_3$	$-i_2$	i_1	-1	$-\varepsilon_7$	$-\varepsilon_6$	ε_5	ε_4	$-\varepsilon_3$	$-\varepsilon_2$	ε_1	i_0
i_0	i_0	$-\varepsilon_1$	$-\varepsilon_2$	$-\varepsilon_3$	$-\varepsilon_4$	$-\varepsilon_5$	$-\varepsilon_6$	$-\varepsilon_7$	-1	i_1	i_2	i_3	i_4	i_5	i_6	i_7
ε_1	ε_1	i_0	ε_3	$-\varepsilon_2$	ε_5	$-\varepsilon_4$	$-\varepsilon_7$	ε_6	i_1	1	$-i_3$	i_2	$-i_5$	i_4	i_7	$-i_6$
ε_2	ε_2	$-\varepsilon_3$	i_0	ε_1	ε_6	ε_7	$-\varepsilon_4$	$-\varepsilon_5$	i_2	i_3	1	$-i_1$	$-i_6$	$-i_7$	i_4	i_5
ε_3	ε_3	ε_2	$-\varepsilon_1$	i_0	ε_7	$-\varepsilon_6$	ε_5	$-\varepsilon_4$	i_3	$-i_2$	i_1	1	$-i_7$	i_6	$-i_5$	i_4
ε_4	ε_4	$-\varepsilon_5$	$-\varepsilon_6$	$-\varepsilon_7$	i_0	ε_1	ε_2	ε_3	i_4	i_5	i_6	i_7	1	$-i_1$	$-i_2$	$-i_3$
ε_5	ε_5	ε_4	$-\varepsilon_7$	ε_6	$-\varepsilon_1$	i_0	$-\varepsilon_3$	ε_2	i_5	$-i_4$	i_7	$-i_6$	i_1	1	i_3	$-i_2$
ε_6	ε_6	ε_7	ε_4	$-\varepsilon_5$	$-\varepsilon_2$	ε_3	i_0	$-\varepsilon_1$	i_6	$-i_7$	$-i_4$	i_5	i_2	$-i_3$	1	i_1
ε_7	ε_7	$-\varepsilon_6$	ε_5	ε_4	$-\varepsilon_3$	$-\varepsilon_2$	ε_1	i_0	i_7	i_6	$-i_5$	$-i_4$	i_3	i_2	$-i_1$	1

Other relations:

$$e^{\varepsilon_n \alpha} = \cosh\ \alpha + \varepsilon_n (\sinh\ \alpha)$$

$$\ln \varepsilon_n = \frac{\pi}{2}(i_0 - i_n)$$

$$\varepsilon_n^{\alpha} = \frac{1}{2}[(1 - \varepsilon_n) + (1 + \varepsilon_n) e^{-\pi i_n \alpha}]$$

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