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Gravity on Four Dimensional Euclidean Spacetime

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Whereas electromagnetic, weak, and strong forces appear to be rooted in Minkowski space-time, gravitation is described through a generally covariant metric. Past approaches to derive Einstein gravity on flat space-time indicated that such an underlying non-covariant background would be unobservable. This article uses this freedom and proposes four dimensional Euclidean space-time as generic background metric for the gravitational force. A program will be offered to guarantee both relativity and equivalence of energy and masses within a defined scope. The required "heavy" Lorentz transformation for the gravitational force will - in the classical view - violate the weak equivalence principle for masses that are not in the same frame of reference. Not taking self-interaction of the gravitational field into account, equivalence to the linearized field equations as obtained from covariant General Relativity will be shown. Gravitational redshift, the event horizon of a black hole, planetary perihelion shift, and deflection of a fast moving body or light will be calculated to indicate feasibility of this approach. A beam focusing concern for (anti-)proton accelerators ($E \gtrsim 1$TeV) will be raised. Transition to quantum gravitation will be executed in the example of spin 1/2 particles. General expectations towards a quantum theory of gravitation will be compared to the free particle wave functions, propagator in energy-momentum and space-time coordinates, and the differential cross section for gravitational Coulomb scattering in lowest order. Implications for the early universe, neutrino behavior, Yang-Mills instantons, and unified forces will be suggested.

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I. INTRODUCTION

Measuring a potential violation of the equivalence principle (VEP) between inert and heavy masses of a test body, as well as, testing the inverse square law of gravitational force is an ongoing experimental challenge that refines allowed parameter regimes for theories aiming to extend gravity (for recent high-precision experiments on VEP see e.g. [1, 2]; for inverse square law see e.g. [3–5]; falling terrestrial gyroscopes have been evaluated e.g. in [6, 7]; lunar ranging data in [8–12], and other solar system tests in [13]; solar neutrinos may be an exception [39]).

Planetary tests are evaluated against a covariant space-time metric obtained from Einstein's General Relativity theory (GRT), and test bodies in terrestrial experiments are typically moving slowly with respect to each other and the lab frame ($v \ll c$). It thus does not contradict experimental findings if one proposes gravity as classical force on a flat background metric without curvature, yielding VEP only when two attracting bodies are in relative motion, with degree of violation increasing with higher speed differences, as long as, the resulting formalism can be proven to be equivalent to General Relativity.

A common approach without VEP originates on Minkowski metric and a proposed gravitational exchange boson of spin 2. With different starting points, [14] (minimized action), [15] (S matrix), and [16] (projection operator, but also note [40]) show equivalence with the linearized field equations as obtained from GRT. Quantum gravitational effects can then be calculated taking self-coupling of the field into account. While the long wavelength structure of the resulting force is in accordance with classical GRT [17], one inevitably runs into problematic short wavelength divergences (see e.g. [18] for an elaborate discussion, or [19] for a detailed overview).

Aside from the quantum gravitational difficulties to go from linearized to covariant GRT through graviton self-interaction there is strong evidence that such a program exists [41]. In order to prove equivalence of an alternative theory of gravitation with General Relativity it will thus be sufficient to show that without self-coupling of the field one obtains linearized GRT.

Already in 1956 a possible VEP was indicated by Kraichnan [20] when examining strong field behavior of the spin 2 formalism. Deser finds in [21] that one is not bound to the (unobservable [22]) Minkowski flat space in order to derive generally covariant Einstein action through "consistent self-coupling requirements, from the linear graviton action". An attempt will therefore be made to build gravitation on a four dimensional Euclidean background instead, exhibiting VEP in the classical sense but not in violation of current experimental findings. Since the difference between Euclidean and Minkowski space-time becomes stronger with increasing relative speeds, this could relate to the problems with spin 2 short wavelength behavior mentioned above, and linear spin 2 graviton coupling in general [21].

II. BASE FORMALISM

While electromagnetic, weak, and strong interaction originate on Minkowski metric, the gravitational force

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will now be presumed to originate on four dimensional Euclidean space-time. Using m and m' as inert and heavy rest mass of a body respectively, it is proposed ($c = 1$):

$$\begin{aligned} m^2 &= E^2 - |\vec{p}|^2 \\ m'^2 &= E^2 + |\vec{p}|^2 \end{aligned} \tag{1}$$

For test bodies moving slowly with respect to each other and the lab frame ($|\vec{p}| \ll E$) we have $m'/m \approx 1$ as experimental data demands. The degree of VEP in the classical sense can be quantified e.g. by using speed $|\vec{v}| = |\vec{p}|/E$:

$$\frac{m'}{m} = \sqrt{\frac{1 + |\vec{v}|^2}{1 - |\vec{v}|^2}} \tag{2}$$

A consistent formalism will be guaranteed by following a *Natural Alignment of Elementary Equations* (*NatAliE* equations) program: While Einstein's Special Relativity theory (SRT) governs non-gravitational effects, a similar formalism will be adapted with as little as possible modifications for purely gravitational effects. For naming purposes physical quantities and effects from SRT will be called "classical" or "inert", and their gravitational counterparts "heavy".

The *NatAliE* program adapts the principle of relativity from Special Relativity. What could be interpreted as a regression from General Relativity only frames of reference will assumed to be equivalent when they are in non-accelerated relative motion.

A needed heavy Lorentz transformation warrants conservation of heavy properties (in particular heavy mass $m' = \sqrt{E^2 + |\vec{p}|^2}$) for frames of reference that are moving with respect to each other at a relative speed $\vec{v}$. If one defines heavy invariant speed $u'_\mu = (u'_0, u'_i)$ with $\mu = (0, 1, 2, 3)$ and $i = (1, 2, 3)$ as

$$\begin{aligned} u'_0 &:= \frac{1}{\sqrt{1 + |\vec{v}|^2}} \\ u'_i &:= \frac{v_i}{\sqrt{1 + |\vec{v}|^2}} \end{aligned} \tag{3}$$

and $|u'_i| := \sqrt{u'^2_1 + u'^2_2 + u'^2_3}$, the following transformation from $dx = (dt, d\vec{x})$ into $dx' = (dt', d\vec{x}')$ coordinates satisfies this need:

$$\begin{aligned} \lambda_{\text{Nat.E.}} &:= \begin{pmatrix} u'_0 & |u'_i| & 0 & 0 \\ -|u'_i| & u'_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\ dx' &= \lambda_{\text{Nat.E.}} dx \end{aligned} \tag{4}$$

In the chosen representation only one spacial component of dx is modified.

The following properties of (4) are shared with classical Lorentz transformation:

1. The transformation is linear and can be represented by a 4×4 matrix.
2. The determinant of $\lambda_{\text{Nat.E.}}$ is $+1$.
3. Relative speed $\vec{v}$ is the only variable.
4. Coordinates are changed in one spatial direction only. Coordinates in the orthogonal plane remain unchanged.

The following differences exist with respect to classical Lorentz transformation:

1. Factors $\sqrt{1 - |\vec{v}|^2}$ are replaced with $\sqrt{1 + |\vec{v}|^2}$.
2. Equation (4) is a true two dimensional rotation in space and time, whereas in the classical case space and time are skewed.
3. The direction of $|u'_i|$ is generally not parallel to the 1-component of dx. It will later show that the 1-component of dx (in above representation) is to be put in direction of the connecting vector between each two attracting masses, independently from the direction of their relative motion $|u'_i|$.

This allows to identify (4) as heavy Lorentz transformation consistent with the proposed alignment program (*NatAliE*) since it conserves heavy mass with as little as needed modifications from the classical counterpart.

Gravitation will be evaluated between each two attracting heavy masses individually. The resulting gravitational forces within many masses or energy density fields will then be obtained through linear superposition.

Similar to classical length contraction heavy Lorentz transformation yields (heavy) length expansion:

$$r = r' \sqrt{1 + |\vec{v}|^2} \tag{5}$$

The measurable r can be interpreted as apparent distance between two attracting heavy masses. The distance effective for gravitational force r' appears expanded to the observer.

There is also heavy time acceleration which one can identify by looking at heavy invariant time $dT := \sqrt{dt^2 + |d\vec{x}|^2}$. It is not obvious how a purely gravitational clock could be built to measure dT. This would be essential to understanding heavy invariant time itself. One may speculate that the expanding universe is such a clock, however, cosmological effects will not be treated within this paper.

III. EQUIVALENCE TO LINEARIZED GENERAL RELATIVITY

Both heavy length expansion (5) and heavy mass (2) can be applied to a pseudo-static $1/r'$ potential with central charge m' ($G=1$):

$$\begin{aligned} \phi_{\rm s} &= \frac{m'}{r'} \\ &= \frac{m}{r}\frac{1+|\vec{v}|^2}{\sqrt{1-|\vec{v}|^2}} \end{aligned} \qquad (6)$$

This is the Newton-like gravitational potential at distance r to a charge with inert mass m and relative speed $|\vec{v}|$ to the observer. Obviously, $\phi_{\rm s}$ is static only in approximation for a slow observer ($|\vec{v}|\,\Delta t/r \ll 1$).

The following will show that without taking self-coupling of the gravitational field into account, SRT generalization of $\phi_{\rm s}$ is equivalent to the linearized field equations obtained from General Relativity.

The metric tensor $g_{\mu\nu}$ will be written as

$$g_{\mu\nu} := 1 + h_{\mu\nu}$$

and heavy mass tensor $M'_{\mu\nu}$ as

$$M'_{\mu\nu} \;:=\; \rho' u_\mu u_\nu \qquad (7)$$

with heavy mass density ρ' and classical invariant speed $u_\mu = g_{\mu\nu}u^\nu := g_{\mu\nu}\gamma\,(1,\vec{v})^\nu$ (using $\gamma = 1/\sqrt{1-|\vec{v}|^2}$). Hydrostatic pressure contribution $P_{\mu\nu}$ to the energy-momentum tensor $T_{\mu\nu} = M_{\mu\nu}+P_{\mu\nu}+...$ can be quantified through kinetic gas theory from mass-momentum distributions and therefore will not be handled here. This simplifies the following calculation without loss of generality.

Keeping in mind that the heavy mass tensor $M'_{\mu\nu}$ contains a heavy quantity ρ' subject to change through the *NatAliE* program, Newton gravity can be expressed as

$$\triangle h'_{00} = 8\pi M'_{00} \qquad (8)$$

and generalized for non-accelerated frames of reference

$$\Box h'_{\mu\nu} \;=\; -8\pi M'_{\mu\nu} \qquad (9)$$

which is solved through retarded potentials:

$$h'_{\mu\nu}\,(t,\vec{x}') \;=\; -2\int d^3r'\,\frac{M'_{\mu\nu}\,(t-|\vec{r}'-\vec{x}'|\,,\vec{r}')}{|\vec{r}'-\vec{x}'|} \qquad (10)$$

Use of metric $d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu$ allows to separate motion of the observer $dx^\mu := (dt, d\vec{x})$ from the motion field u^μ of the heavy charges ρ'. However, the heavy quantity ρ' will need to be evaluated using *total* relative speed between observer and each component of the source field. The same is the case for length-expanded distances $|\vec{r}'-\vec{x}'|$.

Therefore, ρ' will be described through a series of point masses $m'_{(l)}$ where each member has constant $u^\mu_{(l)}$ (in general, this is an infinite sum over infinitesimal addends):

$$\rho' \;:=\; \sum_l m'_{(l)} \qquad (11)$$

Since (9) is linear in $h'_{\mu\nu}$ and ρ' one can calculate $h'_{\mu\nu(l)}$ for each $m'_{(l)}$ and receive the total metric through superposition $h'_{\mu\nu} = \sum_l h'_{\mu\nu(l)}$.

For each member $m'_{(l)}$ in this series coordinates will be chosen for which $u^\mu_{(l)} = (1,0,0,0)$ and $m'_{(l)}$ is located at $\vec{x}' = (0,0,0)$. The total relative speed $|\vec{v}_{(l)}|$ between charge and observer will then be $|d\vec{x}|\,/dt$ of the observer. Equation (10) yields only for $h'_{00(l)}$ a non-zero value:

$$\begin{aligned} h'_{00(l)}\,(t,\vec{x}') &= -2\frac{m'_{(l)}\,(t-|\vec{x}'|\,,0)}{|\vec{x}'|} \\ h'_{\mu\nu(l)} &= -2\frac{m'_{(l)}}{|\vec{x}'|}\begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} \end{aligned} \qquad (12)$$

To simplify appearance the time-dependence $m'_{(l)}\,(t-|\vec{x}'|\,,0)$ of the gravitational sources in not spelled-out anymore.

Replacing heavy mass $m'_{(l)}$ according to (2) with inert mass $m_{(l)}$

$$m'_{(l)} = m_{(l)}\sqrt{\frac{1+|\vec{v}_{(l)}|^2}{1-|\vec{v}_{(l)}|^2}}$$

and distance effective for gravitation $|\vec{x}'|$ per (5) with length expanded distance $|\vec{x}|$

$$|\vec{x}'| = \frac{|\vec{x}|}{\sqrt{1+|\vec{v}_{(l)}|^2}}$$

yields:

$$\begin{aligned} h'_{00(l)}\,(t,\vec{x}) &= -2\frac{m_{(l)}}{|\vec{x}|}\frac{1+|\vec{v}_{(l)}|^2}{\sqrt{1-|\vec{v}_{(l)}|^2}} \\ &= \left[1+\frac{|d\vec{x}|^2}{dt^2}\right]\left(-2\gamma_{(l)}\right)\frac{m_{(l)}}{|\vec{x}|} \\ &:= \left[1+\frac{|d\vec{x}|^2}{dt^2}\right]\left(-2\gamma_{(l)}\right)\phi_{(l)} \end{aligned} \qquad (13)$$

The abbreviation $\phi_{(l)}\,(t,|\vec{x}|) := m_{(l)}\,(t-|\vec{x}|\,,0)\,/\,|\vec{x}|$ can be identified as retarded Newton-like potential of a point mass $m_{(l)}$.

Due to the special choice of coordinates $|d\vec{x}|$ and $|dt|$ are the observer's progression in space and time respectively. This allows to reorder the metric coefficients ($dt \neq 0$):

$$\begin{aligned} d\tau^2 &= (1+h'_{00})\,dt^2 - |d\vec{x}|^2 \\ &= \left\{1+\left[1+\frac{|d\vec{x}|^2}{dt^2}\right]\left(-2\gamma_{(l)}\right)\phi_{(l)}\right\}dt^2 - |d\vec{x}|^2 \\ &= \left\{1-2\gamma_{(l)}\phi_{(l)}\right\}dt^2 - \left\{1+2\gamma_{(l)}\phi_{(l)}\right\}|d\vec{x}|^2 \end{aligned} \qquad (14)$$

The quadratic $|d\vec{x}|^2$ terms in $h'_{00(l)}$ are now associated with the corresponding length element of the metric.

The new metric coefficients are interpreted as

$$\begin{aligned} h_{\mu\nu(l)}(t,\vec{x}) &= -2\gamma_{(l)}\phi_{(l)}\delta_{\mu\nu} \\ &= -2\frac{\gamma_{(l)}m_{(l)}\,(t-|\vec{x}|\,,0)}{|\vec{x}|}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} \end{aligned} \quad (15)$$

which solves the differential equation

$$\Box h_{\mu\nu(l)} = -8\pi\gamma_{(l)}m_{(l)}\,(t,0)\,\delta_{\mu\nu} \quad (16)$$

In order to execute the sum over all point masses $m_{(l)}$ to receive an arbitrary inert mass density $\rho = \sum_l m_{(l)}$ the $\delta_{\mu\nu}$ must be replaced with a tensor. This warrants validity of physical law in all non-accelerated frames of reference.

If one writes $\lambda_{\mu\nu}$ for the classical Lorentz transformation in 4×4 matrix representation, the expression

$$\lambda^2_{\mu\nu} := \left(\lambda\lambda^{\mathrm{T}}\right)_{\mu\nu} \equiv \left(\frac{\partial x^\rho}{\partial x'^\mu}\right)\left(\frac{\partial x^\sigma}{\partial x'^\nu}\right)\delta_{\rho\sigma} \quad (17)$$

is the desired tensor by definition. The total metric $h_{\mu\nu}$ can then be obtained through linear superposition of all $m_{(l)}$ contributions:

$$\Box h_{\mu\nu} = -8\pi\gamma\rho\lambda^2_{\mu\nu} \quad (18)$$

This can be written in familiar form using the inert mass tensor $M_{\mu\nu} := \rho u_\mu u_\nu$ (see appendix A) as

$$\Box h_{\mu\nu} = -16\pi\gamma\left(M_{\mu\nu} - \frac{1}{2}M_\xi^{\ \xi}\eta_{\mu\nu}\right) \quad (19)$$

and identified as linearized field equations obtained from the low energy-momentum density limit of General Relativity, multiplied with a field $\gamma = u^0 = 1/\sqrt{1-|\vec{v}|^2}$. In the GRT approximation for low velocities in the source field, however, an additional factor γ is insignificant ($u_\mu u_\nu$ is already of the order γ^2).

It is concluded that the immediate generalization of Newton gravity for non-accelerated frames of reference

$$\Box h'_{\mu\nu} = -8\pi M'_{\mu\nu}$$

is equivalent to linearized GRT if one follows the *NatAliE* program and replaces heavy mass with inert mass, corrects for length expansion, and re-orders the resulting metric coefficients.

Similar to the approach first proposed by Gupta [23–25] to lead from linearized to covariant relativity by executing an infinite sum of Lagrangians one could argue the following for the *NatAliE* equations: Heavy mass $m' = \sqrt{E^2+|\vec{p}|^2}$ generates a gravitational field which increases total energy E, thus in return increasing its own heavy mass, and so on. Like the Gupta sum, however, it is not obvious how to prove that this directly (without further assumptions) and exactly (without corrections, adjustments, or approximations) leads to generally covariant relativity. Interestingly enough, we currently assume that if such a procedure exist it must *uniquely* lead to GRT (as mentioned in the introduction of this paper, [22–28]).

IV. CLASSICAL EXAMPLES

While derivation of (19) is not taking self-interaction of the gravitational field into account it is important to note that otherwise no approximations have been made. This is in contrast to the common linearized GRT which approximates for weak energy-momentum densities in general. A claim is therefore made that due to the additional factor γ equation (19) exactly separates field self-coupling from the force generated by the sources, at any energy-momentum density therein, including $\rho\to\infty$ and $|u^\mu|\to\infty$.

To support this claim the following example calculations include gravitational redshift and event horizon of a black hole, which can both be obtained from Schwarzschild metric and *NatAliE* without approximation, in contrast to results from linearized GRT central force metric.

We expect to have in Euclidean (*NatAliE*) space $2\pi r$ for circumference of a circle around a central mass M at distance r. In Schwarzschild metric

$$ds^2 = \left(1-\frac{2M}{r}\right)dt^2 - \left(1-\frac{2M}{r}\right)^{-1}dr^2 - r^2 d\Omega^2 \quad (20)$$

with $d\Omega^2 := d\theta^2 + \sin^2\theta d\varphi^2$ this r coordinate is uniquely identified through the distance of one revolution around M being $2\pi r$. The radial term

$$\left(1-\frac{2M}{r}\right)^{-1}dr^2$$

quantifies (and separates) self-coupling of the gravitational field in this scenario. If an experiment doesn't require distance measurement in radial direction, results expressed through the r coordinate must be identical in Schwarzschild metric and *NatAliE* equations calculations.

A. Gravitational Redshift

When light moves through gravitational fields its energy $E = |\vec{p}| = h\nu$ changes due to equivalence of heavy mass, gravitational energy, and inner energy of a photon (strong principle of equivalence).

A photon with energy $E_0 = h\nu_0$ and heavy mass $m'_0 = \sqrt{E_0^2+|\vec{p}_0|^2} = \sqrt{2}E_0$ at distance r' to a static point mass M will have $E_\infty = h\nu_\infty$ if it completely escapes the gravitational field:

$$\begin{aligned} E_\infty &= E_0 - \frac{m'M}{r'} \\ &= E_0\left(1 - \frac{\sqrt{2}M}{r'}\right) \end{aligned} \tag{21}$$

The distance r' effective for gravity appears length expanded to the observer:

$$r' = \frac{r}{\sqrt{1+|\vec{v}|^2}} = \frac{r}{\sqrt{2}} \tag{22}$$

We measure

$$\begin{aligned} E_\infty &= E_0\left(1 - \frac{2M}{r}\right) \\ \nu_\infty &= \nu_0\left(1 - \frac{2M}{r}\right) \end{aligned} \tag{23}$$

in accordance with the experiment and General Relativity (for this and the following examples see e.g. [29] or many other introductions into Einstein gravity).

Radius r is hereby identical to the r coordinate in Schwarzschild metric, as discussed above. Even though the spherical symmetric solution of linearized GRT

$$ds^2 = \left(1 - \frac{2M}{r_{\text{lin}}}\right)dt^2 - \left(1 + \frac{2M}{r_{\text{lin}}}\right)|d\vec{r}_{\text{lin}}|^2 \tag{24}$$

(with $|d\vec{r}_{\text{lin}}|^2 := dr_{\text{lin}}^2 + r_{\text{lin}}^2 d\Omega^2$) seems to yield the same result from the $(1 - 2M/r_{\text{lin}})\,dt^2$ term, the r_{lin} coordinate therein is different and the obtained redshift thus an approximation.

B. Event Horizon of a Black Hole

The speed v of a body m needed to escape the gravitational potential of a central charge M is:

$$\begin{aligned} E_{\text{kin}} &\geq -E_{\text{grav}} \\ \frac{m}{\sqrt{1-v^2}} - m &\geq \frac{Mm'}{r'} \\ m\frac{1-\sqrt{1-v^2}}{\sqrt{1-v^2}} &\geq \frac{Mm}{r}\frac{1+v^2}{\sqrt{1-v^2}} \\ r &\geq M\frac{1+v^2}{1-\sqrt{1-v^2}} \end{aligned} \tag{25}$$

The closest distance to escape from, the event horizon, is at Schwarzschild radius $r = 2M$ when traveling radially with $v = 1$. It does not depend on the escaping body's inert mass m and is also valid for light.

It is noted, however, that (25) cannot be directly compared to Schwarzschild metric for speeds $v < 1$ since v then varies in different frames of reference. Radial distance measurement would be required to determine v for the observer, which cannot be executed without taking self-interaction of the field into account.

C. Perihelion Shift

A satellite with inert mass m, heavy mass m', total energy E, and angular momentum L orbits a large mass $M \gg m$ which is assumed to be at rest and small in size compared to the distance r' between M and m. The satellite m is moving at speeds $v := |\vec{v}| \ll 1$. E and L are constant. These approximations correspond e.g. to planets orbiting our sun.

The sum of kinetic and gravitational energy is:

$$\begin{aligned} E &= E_{\text{kin}} - \frac{Mm'}{r'} \\ &= E_{\text{kin}} - \frac{Mm}{r}\frac{1+v^2}{\sqrt{1-v^2}} \end{aligned} \tag{26}$$

Taking only first order terms in v^2 into account approximates $(1+v^2)/\sqrt{1-v^2} \approx 1 + 2/3v^2$ and

$$E \approx \frac{1}{2}mv^2\left(1 - \frac{3M}{r}\right) - \frac{Mm}{r} \tag{27}$$

which can be written in polar coordinates in the plane of motion (ϕ, r) as

$$\begin{aligned} E &= \frac{m}{2}\left(\dot{r}^2 + \frac{L^2}{m^2r^2}\right)\left(1 - \frac{3M}{r}\right) - \frac{Mm}{r} \\ \dot{r}^2 &= \frac{2}{m}\left(1 - \frac{3M}{r}\right)^{-1}\left(E + \frac{Mm}{r}\right) - \frac{L^2}{m^2r^2} \end{aligned} \tag{28}$$

For a planet orbiting our sun we get in good approximation$(1 - 3M/r)^{-1} \approx 1 + 3M/r$. Defining M^* and L^* as

$$\begin{aligned} M^* &:= M + \frac{3EM}{m} \\ L^* &:= \sqrt{L^2 - 6M^2m^2} \end{aligned} \tag{29}$$

allows to separate (28) into terms of r:

$$\begin{aligned} \dot{r}^2 &= \frac{2E}{m} + \frac{2}{r}\left(M + \frac{3EM}{m}\right) - \frac{L^2 - 6M^2m^2}{m^2r^2} \\ &= \frac{2E}{m} + \frac{2M^*}{r} - \frac{L^{*2}}{m^2r^2} \end{aligned} \tag{30}$$

This has the same form as the classical equation of motion of a body with mass m and angular momentum L^* in a static $1/r$ potential with central charge M^*.

Classical angular velocity $(d/dt)\,\phi_{\text{cl}}$ compared to *NatAliE*'s approximation $(d/dt)\,\phi_{\text{N}}$ is:

$$\begin{aligned} \frac{d}{dt}\phi_{\text{N}} &= \frac{L}{mr^2} = \frac{\sqrt{L^{*2} + 6M^2m^2}}{mr^2} \\ &= \left(\frac{d}{dt}\phi_{\text{cl}}\right)\sqrt{1 + \frac{6M^2m^2}{L^{*2}}} \end{aligned} \tag{31}$$

If a classical body with angular momentum L^* would have done a full revelation $\phi_{\text{full,cl}} = 2\pi$ we have from

NatAliE

$$\begin{aligned}\phi_{\rm full,N} &= \phi_{\rm full,cl}\sqrt{1+\frac{6M^2m^2}{L^{*2}}}\\ &\approx 2\pi+\frac{6\pi M^2m^2}{L^{*2}}\end{aligned} \qquad (32)$$

as demanded by experimental results and GRT calculations under similar approximations.

D. Deflection of a Fast Moving Body or Light

A small deflection angle is calculated in the (x, y) plane on a test body with inert mass m, heavy mass m', incoming speed $v = (v_x, 0, 0)$, and impact parameter b', which travels through a static central force field generated by M. The change in y component of speed v_y is small ($v_y \ll v_x$) and $v = \sqrt{v_x^2 + v_y^2}$ assumed to be constant.

$$\begin{aligned}F_r &= \frac{Mm'}{r'^2}\\ \frac{d}{dt}p_y &= F_r\left(\widehat{e}_r\widehat{e}_y\right) = \frac{Mm'}{r'^2}\left(\widehat{e}_r\widehat{e}_y\right)\end{aligned} \qquad (33)$$

The projection of $\widehat{e}_r$ onto the y-axis can be estimated as $(\widehat{e}_r\widehat{e}_y) \approx b'/r'$ for small deflections:

$$\begin{aligned}\frac{d}{dt}\frac{mv_y}{\sqrt{1-v^2}} &= \frac{Mm'b}{r'^3}\\ \frac{d}{dt}v_y &= \frac{Mm'}{m}\sqrt{1-v^2}\frac{b'}{r'^3}\\ &= \frac{Mm'}{m}\sqrt{1-v^2}\frac{b'}{\left(v^2t^2+b'^2\right)^{3/2}}\\ &= \frac{M}{b'^2}\sqrt{1+v^2}\frac{1}{\left(\left(v/b'\right)^2t^2+1\right)^{3/2}}\end{aligned} \qquad (34)$$

The last step used $m' = m\sqrt{1+v^2}/\sqrt{1-v^2}$ (2).

Total speed in y direction after passing the central mass then becomes:

$$\begin{aligned}\Delta v_y &= \frac{M}{b'^2}\sqrt{1+v^2}\cdot\int_{-\infty}^{+\infty}\frac{dt}{\left(\left(v/b'\right)^2t^2+1\right)^{3/2}}\\ &= \frac{M}{b'^2}\sqrt{1+v^2}\frac{2b'}{v}\\ &\approx \frac{2M}{vb}\left(1+v^2\right)\end{aligned} \qquad (35)$$

Hereby the impact parameter b' was approximated by the length-expanded observable $b = b'\sqrt{1+v^2}$ (5).

The deflection angle $\tan\Theta = \Delta v_y/v$ is then:

$$\Theta \approx \frac{\Delta v_y}{v} = \frac{2M}{v^2b}\left(1+v^2\right) \qquad (36)$$

The same angle is obtained through General Relativity calculations.

E. A Beam Focusing Concern at High Energy Proton Accelerators

The distance d a highly accelerated ($v \approx c$, using SI units in this section only) particle travels until it falls a distance f under the influence of gravitation is

$$d = \sqrt{\frac{2fc^2}{g'}} \qquad (37)$$

(with g' the effective gravitational acceleration of our earth). If one were not to correct for effects from *NatAliE* and assume $g' = g \approx 9.81\,\mathrm{m/s}$ it would e.g. take $d \approx 1355\,\mathrm{m}$ in order to fall $f = 1\,\text{Å}$. The effect of gravitation could be neglected.

Following the *NatAliE* program, however, one obtains g' by correcting the earth's acceleration for gravitational length expansion (5) and heavy mass increase (2). The observed distance between particle and each gravitational source mass m appears length expanded to an observer in Minkowski space-time by a factor $\sqrt{1+v^2/c^2} \approx \sqrt{2}$ and its heavy mass m' is increased to

$$\begin{aligned}\frac{m'}{m} &= \frac{\sqrt{1+v^2/c^2}}{\sqrt{1-v^2/c^2}} = \frac{E_{\rm kin}}{E_{\rm rest}}\sqrt{1+v^2/c^2}\\ &\approx \frac{E_{\rm kin}}{E_{\rm rest}}\sqrt{2}\end{aligned} \qquad (38)$$

This yields:

$$g' \approx \frac{2E_{\rm kin}}{E_{\rm rest}}g \qquad (39)$$

$$d \approx \sqrt{\frac{fc^2}{g}\frac{E_{\rm rest}}{E_{\rm kin}}} \qquad (40)$$

The distance d for the particle to fall a certain distance f decreases with increasing kinetic energy [42].

In order to estimate a potential impact on current and future particle accelerators the distance it takes for protons ($E_{\rm rest} \approx 0.938\,\mathrm{GeV}$) to fall $1\,\text{Å}$ is calculated when moving at $E_{\rm kin} = 980\,\mathrm{GeV}$ (Tevatron at Fermilab) and $E_{\rm kin} = 7000\,\mathrm{GeV}$ (planned for LHC at CERN). According to *NatAliE* these distances become

$$\begin{aligned}d\big|_{1\,\text{Å},\,980\,\mathrm{GeV}} &\approx 29\,\mathrm{m}\\ d\big|_{1\,\text{Å},\,7000\,\mathrm{GeV}} &\approx 11\,\mathrm{m}\end{aligned}$$

respectively and are well within the distance range of guiding components at these accelerators. With proton particle bundle diameters of just a few Å it could be of concern that they may be entering focusing and cooling elements out of their optimal region of operation, resulting in a less focused and hotter beam. This may be unexpected for protons which can generally be focused and cooled efficiently due to their high mass at rest (as compared to electrons). Whether or not this has a detrimental effect on the accelerator's luminosity depends on the particular component configuration in use and could be evaluated using simulations which take *NatAliE* corrections into account.

V. QUANTUM GRAVITATION

The mass-energy-momentum relation for heavy mass

$$m'^2 \;=\; E^2 + |\vec{p}|^2 \tag{41}$$

can be quantized in direct analogy to the classical inert mass relation $m^2 = E^2 - |\vec{p}|^2$. The following calculations are therefore executed to give supporting evidence for validity of the *NatAliE* equations program for quanta. Discussion is limited to basic properties and behavior of spin $1/2$ particles. Quantization of fields will not be discussed, and gravitation will not be mixed with other forces. This limitation of scope aims at demonstrating plausibility of the fundamental ideas. It also avoids inherent uncertainties when attempting to unify the known forces.

A. The Heavy Dirac Equation

The classical Dirac equation can be obtained from $m^2 = E^2 - |\vec{p}|^2$ through linearization and subsequent operator transition. The same procedure can be followed for *NatAliE* gravitation. Using the following representation of Pauli matrices

$$\begin{aligned} \epsilon &:= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \sigma_y &:= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ \sigma_x &:= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \sigma_z &:= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned} \tag{42}$$

and an outer product

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} aA & aB & bA & bB \\ aC & aD & bC & bD \\ cA & cB & dA & dB \\ cC & cD & dC & dD \end{pmatrix} \tag{43}$$

the following relation linearizes (41)

$$m' = + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \times \epsilon E + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \times \vec{\sigma}\vec{p} \tag{44}$$

(with $\vec{\sigma}\vec{p} := \sigma_x p_1 + \sigma_y p_2 + \sigma_z p_3$). The "+" signs are chosen to later interpret positive and negative energy states as particles and anti-particles respectively, both with positive mass in analogy to the classical case.

Operator transition

$$E \to +i\frac{\partial}{\partial t} := i\partial_0 \qquad p_i \to -i\frac{\partial}{\partial x_i} := -i\partial_i \tag{45}$$

yields the proposed heavy Dirac equation which can be written with use of

$$\begin{aligned} \beta_0 &:= \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \\ \beta_i &:= \begin{pmatrix} 0 & -\sigma_i \\ -\sigma_i & 0 \end{pmatrix} \end{aligned} \tag{46}$$

as:

$$[i\beta_\nu \partial_\nu - m']\,\Psi = 0 \tag{47}$$

(since *NatAliE* operates on four dimensional Euclidean metric all indices are written as lower indices; duplicate Greek indices in a product are summed $\nu = 0,1,2,3$ unless otherwise stated).

Similar to the classical case on Minkowski space-time the sum

$$\frac{1}{2}\left(\beta_\mu\beta_\nu + \beta_\nu\beta_\mu\right) = \delta_{\mu\nu} \tag{48}$$

yields the underlying metric which is Euclidean here. However, the β matrices remain invariant when Hermitian conjugated, i.e.

$$\beta_\mu = \tilde{\beta}_\mu \tag{49}$$

This important property allows to define the adjunct wave function $\tilde{\Psi}$ simply as transpose of the complex conjugated components of Ψ, i.e. $\tilde{\Psi} := (\Psi^*)^{\mathrm{T}}$. It solves the adjunct heavy Dirac equation

$$\tilde{\Psi}\left[-i\beta_\nu \overleftarrow{\partial_\nu} - m'\right] \;=\; 0$$

since $\left(\left[-i\beta_\nu^T \partial_\nu - m'\right]\tilde{\Psi}^{\mathrm{T}}\right)^* = [i\beta_\nu\partial_\nu - m']\,\Psi$ and $\beta = \tilde{\beta}$.

One can validate conservation of probability density $j_\mu := \tilde{\Psi}\beta_\mu\Psi$ as a test for consistency of these definitions:

$$\begin{aligned} \nabla_\mu j_\mu &= \nabla_\mu\left[\tilde{\Psi}\beta_\mu\Psi\right] \\ &= \left(\nabla_\mu\tilde{\Psi}\right)\beta_\mu\Psi + \tilde{\Psi}\beta_\mu\left(\nabla_\mu\Psi\right) \\ &= \nabla_\mu\left(\tilde{\beta}_\mu\Psi\right)^{*\mathrm{T}}\Psi + \tilde{\Psi}\left(\beta_\mu\nabla_\mu\Psi\right) \\ &= \left(\beta_\mu\nabla_\mu\Psi\right)^{*\mathrm{T}}\Psi + \tilde{\Psi}\left(\beta_\mu\nabla_\mu\Psi\right) \\ &= \left(-im'\Psi\right)^{*\mathrm{T}}\Psi + \tilde{\Psi}\left(-im'\Psi\right) \\ &= im'\left(\tilde{\Psi}\Psi - \tilde{\Psi}\Psi\right) = 0 \end{aligned}$$

In general, invariance under Hermitian conjugation for β_μ is an important difference to the classical case and ensures that both particles and anti-particles are treated the same with respect to gravitation.

B. Free Spin 1/2 Particle Solution

Spelled-out explicitely the heavy Dirac equation (47)

$$\begin{pmatrix} -m'+i\partial_0 & 0 & -i\partial_3 & -i\partial_1-\partial_2 \\ 0 & -m'+i\partial_0 & -i\partial_1+\partial_2 & i\partial_3 \\ -i\partial_3 & -i\partial_1-\partial_2 & -m'-i\partial_0 & 0 \\ -i\partial_1+\partial_2 & i\partial_3 & 0 & -m'-i\partial_0 \end{pmatrix} \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{50}$$

has the following four linear independent solutions:

$$\Psi_1^+ := \exp i(\vec{p}\vec{x}-Et) \begin{pmatrix} 1 \\ 0 \\ p_3/(m'+E) \\ (p_1+ip_2)/(m'+E) \end{pmatrix}$$
$$\Psi_1^- := \exp i(\vec{p}\vec{x}-Et) \begin{pmatrix} 0 \\ 1 \\ (p_1-ip_2)/(m'+E) \\ -p_3/(m'+E) \end{pmatrix}$$
$$\Psi_2^+ := \exp i(\vec{p}\vec{x}+Et) \begin{pmatrix} p_3/(m'+E) \\ (p_1+ip_2)/(m'+E) \\ 1 \\ 0 \end{pmatrix}$$
$$\Psi_2^- := \exp i(\vec{p}\vec{x}+Et) \begin{pmatrix} (p_1-ip_2)/(m'+E) \\ -p_3/(m'+E) \\ 0 \\ 1 \end{pmatrix}$$

Using spinors

$$\chi^+ := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi^- := \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{51}$$

this can be written as

$$\begin{aligned} \Psi_1^\pm &= \exp i(\vec{p}\vec{x}-Et) \begin{pmatrix} \chi^\pm \\ [\vec{\sigma}\vec{p}/(m'+E)]\,\chi^\pm \end{pmatrix} \\ \Psi_2^\pm &= \exp i(\vec{p}\vec{x}+Et) \begin{pmatrix} [\vec{\sigma}\vec{p}/(m'+E)]\,\chi^\pm \\ \chi^\pm \end{pmatrix} \end{aligned} \tag{52}$$

and identified as the spin up/down ($\pm$) solutions of particles (Ψ_1) and anti-particles (Ψ_2). Other than the sign before Et in the exponent term, there is not difference in the free particle and anti-particle wave functions in accordance to what one might expect from gravitation and a quantum theory thereof.

C. Free Spin 1/2 Particle Propagator in Space-Time Coordinates

While there will not be an evaluation of Feynman graphs in this paper, the classical QED particle propagator reflects fundamental properties of the underlying Minkowski metric. When expressed in space-time coordinates each section of the light cone must be evaluated separately. Similarly, a heavy spin 1/2 particle propagator on Euclidean space-time should be spherically symmetric in all four dimensions and yield behavior in accordance with expectations towards quantum gravitation.

The free particle propagator $S(x-y)$ with four-vectors x and y (y constant) is defined through:

$$[i\beta_\nu\partial_\nu - m']\,S(x-y) = \delta^4(x-y) \tag{53}$$

In energy-momentum coordinates $p := (E, \vec{p})$

$$\begin{aligned} S(x-y) &= \int \frac{d^4p}{(2\pi)^4} S(p) \exp[-ip_\nu(x_\nu-y_\nu)] \\ \delta^4(x-y) &= \int \frac{d^4p}{(2\pi)^4} \exp[-ip_\nu(x_\nu-y_\nu)] \end{aligned} \tag{54}$$

this becomes:

$$(\beta_\nu p_\nu - m')\,S(p) = 1 \tag{55}$$

Using

$$\begin{aligned} \beta_\mu\beta_\nu p_\mu p_\nu &= \frac{1}{2}[\beta_\mu\beta_\nu p_\mu p_\nu + \beta_\nu\beta_\mu p_\nu p_\mu] \\ &= \frac{1}{2}[(\beta_\mu\beta_\nu + \beta_\nu\beta_\mu)\,p_\mu p_\nu] \\ &= \delta_{\mu\nu} p_\mu p_\nu =: |p|^2 \end{aligned} \tag{56}$$

this is solved by:

$$S(p) = \frac{\beta_\nu p_\nu + m'}{|p|^2 - m'^2} \tag{57}$$

Representation of $S(p)$ is space-time coordinates is then obtained by executing (54):

$$\begin{aligned} S(x-y) &= \int \frac{d^4p}{(2\pi)^4} \frac{\beta_\nu p_\nu + m'}{|p|^2 - m'^2} \exp[-ip_\nu(x_\nu-y_\nu)] \\ &= \frac{i\beta_\nu\partial_\nu + m'}{(2\pi)^4} T(x-y) \end{aligned} \tag{58}$$

The remaining integral

$$T(x-y) := \int d^4p \frac{\exp[-ip_\nu(x_\nu-y_\nu)]}{|p|^2 - m'^2} \tag{59}$$

is spherically symmetric as demanded and only depends on $|p|$, $|x-y|$, and relative angle $\cos\alpha = p_\nu(x_\nu-y_\nu)/(|p|\,|x-y|)$. It will therefore be expressed

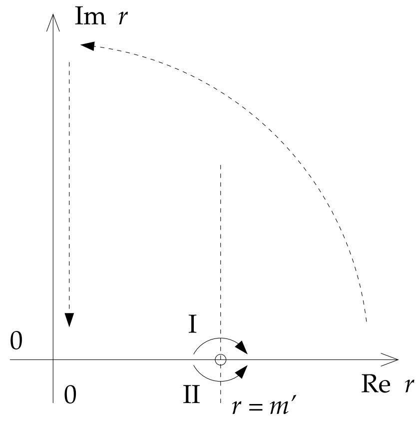

Figure 1: The integral is closed in the first quadrant as indicated. Path I symbolizes integration excluding, path II including the pole at $r = m'$.

in four dimensional spheric coordinates $(r, \alpha, \theta, \phi)$ with direction of $(x - y)$ at $\alpha = 0$, $R := |x - y|$, $r := |p|$, and

$$\begin{aligned} p_0 &:= r \cos\alpha \\ p_1 &:= r \sin\alpha \sin\theta \cos\phi \\ p_2 &:= r \sin\alpha \sin\theta \sin\phi \\ p_3 &:= r \sin\alpha \cos\theta \end{aligned} \tag{60}$$

(θ, ϕ arbitrary) with variable ranges

$$(r, \alpha, \theta, \phi) = (0 \cdots \infty,\ 0 \cdots \pi,\ 0 \cdots \pi,\ 0 \cdots 2\pi)$$

as:

$$\begin{aligned} T(R) &= \int dr\, d\alpha\, d\theta\, d\phi\, r^3 \sin^2\alpha \sin\theta \frac{\exp[-irR\cos\alpha]}{r^2 - m'^2} \\ &= 4\pi \int \frac{dr\, r^3}{r^2 - m'^2} \int d\alpha\, \sin^2\alpha \exp[-irR\cos\alpha] \end{aligned}$$

Substituting $u := \cos\alpha$ allows to identify the integral over α as Bessel function of the first kind of order one, J_1 (for nomenclature see [30] and appendix B):

$$\begin{aligned} &\int_0^\pi d\alpha\, \sin^2\alpha \exp[-irR\cos\alpha] \\ &= \int_{-1}^{1} du \sqrt{1-u^2} \exp[-irRu] = \frac{\pi}{rR} J_1(rR) \end{aligned} \tag{61}$$

Integration over r

$$T(R) = \frac{4\pi^2}{R} \int_0^\infty dr \frac{r^2}{r^2 - m'^2} J_1(rR) \tag{62}$$

requires attention at the pole $r^2 = m'^2$. A closed integral will therefore be executed as indicated in figure 1 excluding and including the residual.

To realize path I (excluding the residual) heavy mass m' is shifted slightly into the negative imaginary plane, $m' \to m' - i\varepsilon$ with $\varepsilon \to 0$. The substitutions

$$\begin{aligned} s &:= rR \\ b &:= iR(m' - i\varepsilon) \end{aligned}$$

bring (62) into a known form and yield McDonald function of order one (K_1) and Hankel function of the second kind of order one, $H_1^{(2)}$:

$$\begin{aligned} T(R)_{\mathrm{I}} &= \frac{4\pi^2}{R^2} \int_0^\infty ds \frac{s^2}{s^2 + b^2} J_1(s) \qquad (Re(b) > 0) \\ &= \frac{4\pi^2}{R^2} [bK_1(b)] \\ &= i\frac{4\pi^2 m'}{R} K_1(im'R) \qquad (\varepsilon \to 0) \\ &= -i\frac{2\pi^3 m'}{R} H_1^{(2)}(m'R) \end{aligned} \tag{63}$$

For integration path I the propagator is then as function of $R = |x - y|$:

$$\begin{aligned} S(R)_{\mathrm{I}} &= \frac{i\beta_\nu\partial_\nu + m'}{(2\pi)^4} \left[-i\frac{2\pi^3 m'}{R} H_1^{(2)}(m'R)\right] \\ &= (\beta_\nu\partial_\nu - im') \frac{m'}{8\pi R} H_1^{(2)}(m'R) \end{aligned} \tag{64}$$

Path II is realized by adding the value of the residual in (62) at $r = m'$ to $T(R)_{\mathrm{I}}$:

$$\begin{aligned} T(R)_{\mathrm{II}} &= T(R)_{\mathrm{I}} + 2\pi i \left[\frac{4\pi^2}{R}\frac{m'^2}{2m'} J_1(m'R)\right] \\ &= i\frac{2\pi^3 m'}{R}\left(-H_1^{(2)}(m'R) + 2J_1(m'R)\right) \\ &= i\frac{2\pi^3 m'}{R} H_1^{(1)}(m'R) \\ &= i\frac{2\pi^3 m'}{R} H_1^{(2)}(-m'R) = -T(-R)_{\mathrm{I}} \end{aligned} \tag{65}$$

This yields:

$$\begin{aligned} S(R)_{\mathrm{II}} &= \frac{i\beta_\nu\partial_\nu + m'}{(2\pi)^4}\left[i\frac{2\pi^3 m'}{R} H_1^{(2)}(-m'R)\right] \\ &= (\beta_\nu\partial_\nu - im') \frac{m'}{8\pi(-R)} H_1^{(2)}(m'(-R)) \\ &= S(-R)_{\mathrm{I}} \end{aligned} \tag{66}$$

Since $R := |x - y|$ is positive by definition the difference between both integration paths $S(R)_{\mathrm{II}} = S(-R)_{\mathrm{I}}$ is merely reflecting this freedom.

It is concluded that (64) represents the free spin 1/2 particle propagator in space-time coordinates

$$S(x - y) = (\beta_\nu\partial_\nu - im') \frac{m'}{8\pi|x - y|} H_1^{(2)}(m'|x - y|) \tag{67}$$

for the choice of positive norms. Four dimensional Euclidean metric is apparent and (67) does not require special consideration with respect to the light cone. The propagator does not distinguish between particles and anti-particles thus satisfying a general expectation towards quantum gravity.

D. Spin 1/2 Coulomb Scattering in Lowest Order

To further indicate plausibility of *NatAliE* equations for quantum gravitation, purely gravitational spin 1/2 Coulomb scattering on a fixed target (Rutherford scattering) will be executed in lowest order. A heavy Dirac equation with field is therefore needed. In order to avoid speculations or statements conflicting with ongoing efforts to find a unified field theory through symmetry transformations on the classical Dirac equation we will restrict ourselves to a static $M/|\vec{x}|$ potential and propose that the intuitive

$$\left(i\beta_\nu\partial_\nu - \beta_0\frac{m'M}{|\vec{x}|} - m'\right)\Psi = 0 \tag{68}$$

is at least a valid approximation for the current calculation. Building a more general and complete quantum gravity framework may well be tied to the challenge of unifying all fundamental forces and is thus out of the scope of this paper.

Calculating a scattering cross section from equations structured like (68) is a well-known procedure and discussed in-depth in many introductory-level QED books (e.g. [31], chapter 5).

The functions

$$\begin{aligned} u_1^\pm(p) &:= \sqrt{\frac{m'+E}{2m'}}\begin{pmatrix}\chi^\pm \\ [\vec{\sigma}\vec{p}/(m'+E)]\,\chi^\pm\end{pmatrix} \\ u_2^\pm(p) &:= \sqrt{\frac{m'+E}{2m'}}\begin{pmatrix}[\vec{\sigma}\vec{p}/(m'+E)]\,\chi^\pm \\ \chi^\pm\end{pmatrix} \end{aligned} \tag{69}$$

are eigenvectors to the heavy Dirac equation's free particle solutions (52) and normalized to $\langle u_j^\pm | u_j^\pm\rangle = 1$ (with $j = 1, 2$). When an incoming particle approaches a static target the distance effective for gravitation appears length expanded. The incoming wave functions $\Psi_j^\pm$ must therefore be normalized to the invariant volume V_0

$$V_0 := \frac{V}{\sqrt{1+|\vec{v}|^2}} = \frac{VE}{m'} \tag{70}$$

and become

$$\Psi_j^\pm = \sqrt{\frac{m'}{EV}}\exp i\left(\vec{p}\vec{x} + (-1)^j Et\right)u_j^\pm(p) \tag{71}$$

respectively.

The probability of taking the particle from the initial into the final state $S_{\mathrm{fi}} = \langle \mathrm{f}|\,S\,|\mathrm{i}\rangle$ for a potential as in (68) is then in lowest order (abbreviating $U := u_{\mathrm{f}}^*\beta_0 u_{\mathrm{i}}$)

$$\begin{aligned} S_{\mathrm{fi}} &= im'\int d^4x\,\left(\Psi_j^\pm\right)_{\mathrm{f}}^*\beta_0\frac{M}{|\vec{x}|}\left(\Psi_j^\pm\right)_{\mathrm{i}} \\ &= -i\frac{m'^2M}{V}\sqrt{\frac{1}{E_{\mathrm{f}}E_{\mathrm{i}}}}U\int d^4x\frac{\exp\left[i\left(p_{\mathrm{f},\nu}-p_{\mathrm{i},\nu}\right)x_\nu\right]}{|\vec{x}|} \\ &= -i\frac{m'^2M}{V}\sqrt{\frac{1}{E_{\mathrm{f}}E_{\mathrm{i}}}}U\frac{8\pi^2}{|\vec{p}_{\mathrm{f}}-\vec{p}_{\mathrm{i}}|^2}\delta\left(E_{\mathrm{f}}-E_{\mathrm{i}}\right) \end{aligned} \tag{72}$$

(the integral yields the same result for particles $j = 1$ and anti-particles $j = 2$ since $\delta(E_{\mathrm{f}} - E_{\mathrm{i}}) \equiv \delta(E_{\mathrm{i}} - E_{\mathrm{f}})$).

In direct comparison to classical electromagnetic (EM) Coulomb scattering of a particle with charge e on a target with charge Ze

$$S_{\mathrm{fi,EM}} = -i\frac{mZe^2}{V}\sqrt{\frac{1}{E_{\mathrm{f}}E_{\mathrm{i}}}}U_{\mathrm{EM}}\frac{8\pi^2}{|\vec{p}_{\mathrm{f}}-\vec{p}_{\mathrm{i}}|^2}\delta\left(E_{\mathrm{f}}-E_{\mathrm{i}}\right) \tag{73}$$

one obtains the gravitational cross section $d\sigma$ by replacing $mZe^2 \to m'^2M$ and $U_{\mathrm{EM}} \to U$:

$$d\sigma = \frac{m'^4M^2}{4|\vec{p}|^4\sin^4(\theta/2)}U^2d\Omega \tag{74}$$

Without spin contribution $U^2 := |u_{\mathrm{f}}^*\beta_0u_{\mathrm{i}}|^2$ this is equal to Newton gravity scattering of classical bodies.

Calculation of U^2 can be simplified by setting the p_3 component of incoming and outgoing particle waves to zero. The effect will then be calculated in the (x_1,x_2) plane without loss of generality, reflecting the freedom of choice of which direction to consider a particle spin "up" or "down":

$$\begin{aligned} u_{\mathrm{f/i}}^+(p) &= \sqrt{\frac{m'+E}{2m'}}\begin{pmatrix}1\\0\\0\\(p_1+ip_2)/(m'+E)\end{pmatrix} \\ u_{\mathrm{f/i}}^-(p) &= \sqrt{\frac{m'+E}{2m'}}\begin{pmatrix}0\\1\\0\\(p_1-ip_2)/(m'+E)\end{pmatrix} \end{aligned}$$

It follows immediately that particle spin remains unchanged:

$$\begin{aligned} \langle +|\,\beta_0\,|-\rangle &= \left(u_{\mathrm{f}}^+\right)^*\beta_0u_{\mathrm{i}}^- \\ &= 0 \\ &= \langle -|\,\beta_0\,|+\rangle \end{aligned} \tag{75}$$

Since the scattering target is fixed this is expected since total spin should be conserved. The contribution is then:

$$\begin{aligned} \langle +|\,\beta_0\,|+\rangle &= \left(u_{\mathrm{f}}^+\right)^*\beta_0u_{\mathrm{i}}^+ \\ &= \frac{m'+E}{2m'}\left(-1+\frac{\left(p_{\mathrm{f},1}-ip_{\mathrm{f},2}\right)\left(p_{\mathrm{i},1}+ip_{\mathrm{i},2}\right)}{(m'+E)^2}\right) \end{aligned} \tag{76}$$

Using momentum $|\vec{p}|$ and scattering angle θ one can write in the (x_1, x_2) plane

$$\begin{aligned} p_{\mathrm{f},1}p_{\mathrm{i},1} + p_{\mathrm{f},2}p_{\mathrm{i},2} &= \vec{p}_\mathrm{f}\vec{p}_\mathrm{i} = |\vec{p}|^2 \cos\theta \\ p_{\mathrm{f},1}p_{\mathrm{i},2} - p_{\mathrm{f},2}p_{\mathrm{i},1} &= (\vec{p}_\mathrm{f} \times \vec{p}_\mathrm{i})_3 = -|\vec{p}|^2 \sin\theta \end{aligned}$$

and get:

$$(p_{\mathrm{f},1} - ip_{\mathrm{f},2})(p_{\mathrm{i},1} + ip_{\mathrm{i},2}) = |\vec{p}|^2 (\cos\theta - i\sin\theta) \quad (77)$$

With no spin-flip possible and $|\langle -|\,\beta_0\,|-\rangle|^2 = |\langle +|\,\beta_0\,|+\rangle|^2$ due to symmetry under spin orientation the total spin contribution U^2 is then:

$$\begin{aligned} U^2 &= |u_\mathrm{f}^* \beta_0 u_\mathrm{i}|^2 = |\langle +|\,\beta_0\,|+\rangle|^2 \\ &= \left| \frac{-(m'+E)^2 + |\vec{p}|^2 (\cos\theta - i\sin\theta)}{2m'(m'+E)} \right|^2 \\ &= \frac{(m'+E)^4 + |\vec{p}|^4 - 2(m'+E)^2 |\vec{p}|^2 \cos\theta}{4m'^2 (m'+E)^2} \quad (78) \end{aligned}$$

With $\cos\theta = 1 - 2\sin^2\theta/2$ and $m'^2 = E^2 + |\vec{p}|^2$ this can be transformed to

$$\begin{aligned} U^2 &= \frac{E^2}{m'^2}\left(1 + \frac{|\vec{p}|^2}{E^2}\sin^2\frac{\theta}{2}\right) \\ &= \frac{1}{1+|\vec{v}|^2}\left(1 + |\vec{v}|^2 \sin^2\frac{\theta}{2}\right) \quad (79) \end{aligned}$$

and together with (74) yields the differential cross section of gravitational spin 1/2 Coulomb scattering on a fixed target:

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{m'^4 M^2}{4|\vec{p}|^4 \sin^4(\theta/2)} \frac{E^2}{m'^2}\left(1 + \frac{|\vec{p}|^2}{E^2}\sin^2\frac{\theta}{2}\right) \quad (80) \\ &= \frac{M^2}{4|\vec{v}|^4 \sin^4(\theta/2)}\left(1+|\vec{v}|^2\right)\left(1 + |\vec{v}|^2\sin^2\frac{\theta}{2}\right) \end{aligned}$$

(expressing heavy through inert mass $m'^2 = m^2\left(1+|\vec{v}|^2\right)/\left(1-|\vec{v}|^2\right)^{-1}$ and $m^2/|\vec{p}|^2 = \left(1-|\vec{v}|^2\right)/|\vec{v}|^2$).

As compared to Newton gravity we have an increased probability for backscattering ($\theta > \pi/2$). Towards the speed of light ($|\vec{v}| \to 1$) the cross section approaches a constant value:

$$\lim_{|\vec{v}|\to 1} \frac{d\sigma}{d\Omega} = \frac{M^2}{2\sin^4(\theta/2)}\left(1 + \sin^2\frac{\theta}{2}\right) \quad (81)$$

This is in qualitative agreement with General Relativity scattering behavior on a black hole which allows incoming bodies or light to be deflected back into the originating direction. Also, particles can be captured at any incoming energy ($|\vec{v}| \to 1$). It is in contrast to electromagnetic behavior (spin 1/2 particle of charge e and inert mass m gets scattered at a static central charge Ze) where the cross section disappears for $E \to \infty$ and backscattering is suppressed:

$$\begin{aligned} \left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{EM}} &= \frac{m^2 Z^2 e^4}{4|\vec{p}|^4 \sin^4(\theta/2)} \frac{E^2}{m^2}\left(1 - \frac{|\vec{p}|^2}{E^2}\sin^2\frac{\theta}{2}\right) \quad (82) \\ &= \frac{Z^2 e^4/m^2}{4|\vec{v}|^4 \sin^4(\theta/2)}\left(1-|\vec{v}|^2\right)\left(1 - |\vec{v}|^2\sin^2\frac{\theta}{2}\right) \end{aligned}$$

Comparing gravitational cross section as obtained through *NatAliE* equations (80) with electromagnetism (82) yields:

$$\begin{aligned} &\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{NatAliE}} \Big/ \left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{QED}} \\ &= \frac{m^2 M^2}{Z^2 e^4} \frac{\left(1+|\vec{v}|^2\right)\left(1+|\vec{v}|^2\sin^2(\theta/2)\right)}{\left(1-|\vec{v}|^2\right)\left(1-|\vec{v}|^2\sin^2(\theta/2)\right)} \\ &= \frac{m'^2 M^2}{Z^2 e^4} \frac{\left(1+|\vec{v}|^2\sin^2(\theta/2)\right)}{\left(1-|\vec{v}|^2\sin^2(\theta/2)\right)} \quad (83) \end{aligned}$$

While the relative strength of the two effects at low energies are still proportional to the relative strength of the underlying forces ($m'^2 M^2/Z^2 e^4$) the behavior changes at very high energies, in particular in the backward ($\theta \approx \pi/2$) direction.

VI. OUTLOOK

Earlier, a potential beam focusing concern for current and future high-energy proton accelerators was raised. It was also noted that it remains to be shown whether quantifying self-coupling of the gravitational field with respect to the *NatAliE* equations program will directly and exactly lead to the General Relativity field equations. This may be essential to understanding how cosmology and heavy invariant time can be interpreted with respect to gravitation on flat Euclidean space-time.

But one may also begin to carefully consider implications for gravity interactions with other forces. Particles at energy levels shortly after the big bang may be subject to notable gravitational backscattering (81) in excess of what may currently be anticipated. The otherwise weakly interacting neutrinos may play a greater role e.g. in slowing dispersion of energy concentrations from today's galaxies into outer space.

One may even speculate that Yang-Mills pseudoparticle solutions [32] needed to avoid a low energy divergence for the strong force could be related to gravitation: Interpreted as vacuum tunneling states between Euclidean and Minkowski metric they may reflect the presence of classical (SRT) and naturally aligned (*NatAliE*) space-time. If one would roughly approximate the strength of measurable effects through the relative strength of gravity with respect to the other forces, this could explain why e.g. an expected electric dipole of a neutron has not been measured yet [33].

Final resolution to these and many other speculations may - as so often now - bring unification of the four known fundamental forces. The expression

$$m^2 m'^2 = E^4 - |\vec{p}|^4 \tag{84}$$

would certainly contain the classical $m^2 = E^2 - |\vec{p}|^2$ and proposed heavy $m'^2 = E^2 + |\vec{p}|^2$. With fundamental mass-energy-momentum relations so closely aligned, the challenge here may be not so much *how* to unify the forces but *why* to chose one way over the other. Simplicity, a low amount of principles and nature constants, and no solutions without an appearance in the experiment (no "ghosts") would certainly be strong arguments.

Appendix A: LINEARIZED GRT ADDENDUM

With an arbitrary rotation in space α every classical Lorentz transformation λ can be brought into the form $\lambda = \alpha\bar{\lambda}\alpha^{-1}$ with

$$\bar{\lambda}_{\mu\nu} := \begin{pmatrix} u_0 & u_1 & 0 & 0 \\ u_1 & u_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

affecting x_1 direction only. Since $\bar{\lambda}^{\mathrm{T}} = \bar{\lambda}$ and $\alpha^{\mathrm{T}} = \alpha^{-1}$ we have $\lambda^{\mathrm{T}} = \left(\alpha\bar{\lambda}\alpha^{-1}\right)^{\mathrm{T}} = \lambda$ and thus $\lambda^2 = \lambda\lambda^{\mathrm{T}} = \alpha\bar{\lambda}\bar{\lambda}\alpha^{-1}$ with

$$\left(\bar{\lambda}\bar{\lambda}\right)_{\mu\nu} \equiv \bar{\lambda}_{\mu\xi}\bar{\lambda}_{\xi\nu} = \begin{pmatrix} u_0^2+u_1^2 & 2u_0u_1 & 0 & 0 \\ 2u_0u_1 & u_0^2+u_1^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Similarly, the inert mass tensor M of a point charge can be written as $M := \alpha\bar{M}\alpha^{-1}$ with

$$\bar{M}_{\mu\nu} = \rho \begin{pmatrix} u_0^2 & u_0u_1 & 0 & 0 \\ u_0u_1 & u_1^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and orientation-independent trace $\bar{M}_\xi^{\ \xi} = \bar{M}_{\xi\nu}\eta^{\nu\xi} = \rho\left(u_0^2 - u_1^2\right) = \rho = M_\xi^{\ \xi}$. We then have

$$\begin{aligned} & 2\bar{M}_{\mu\nu} - M_\xi^{\ \xi}\eta_{\mu\nu} \\ &= \rho \begin{pmatrix} 2u_0^2-1 & 2u_0u_1 & 0 & 0 \\ 2u_0u_1 & 2u_1^2+1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\ &= \rho\left(\bar{\lambda}\bar{\lambda}\right)_{\mu\nu} \end{aligned}$$

and can immediately generalize $2M_{\mu\nu} - M_\xi^{\ \xi}\eta_{\mu\nu} = \rho\lambda_{\mu\nu}^2$ since space is isotropic under α.

Appendix B: BESSEL, HANKEL, AND MCDONALD FUNCTIONS

Nomenclature as in [30] has been adapted for this paper. The following definitions and relations are used:

- J_1 is the Bessel function of the first kind of order one:

$$J_1(z) = \frac{z}{\pi}\int_{-1}^{1} du\sqrt{1-u^2}\exp[-iuz]$$

- K_1 is the McDonald function of order one:

$$K_1(z) = \frac{1}{z}\int_0^\infty ds\frac{s^2}{s^2+z^2}J_1(s) \qquad (Re(z) > 0)$$

- $H_1^{(1)}$ and $H_1^{(2)}$ are the Hankel functions of order one of the first and second kind respectively, with the following identities:

$$\begin{aligned} H_1^{(1)}(z) &= -\frac{2}{\pi}K_1(-iz) \\ &= H_1^{(2)}(-z) \\ &= -H_1^{(2)}(z) + 2J_1(z) \end{aligned}$$

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[40] Effects from the linearized GRT are derived by Deser and Laurent ([16]), however, a claim is made that the approach therein would yield generally covariant relativity. This was later put in question by Deser [21].
[41] Kraichnan [26] shows that such formalisms must necessarily be generally covariant field theories. Gupta [23–25] ascertains that the only known physical quantity described by a symmetrical tensor which satisfies a vanishing divergence is the total energy-momentum tensor of a closed system. He suggests an infinite iteration series of terms in the Lagrangian to quantify self-coupling of the field. Thirring [22] executes the first order correction from such a series in spherically symmetric static metric and finds agreement with the corresponding approximation from Schwarzschild coordinates in GRT. Deser then shows [27] that the particular result of this iteration must be of a certain form to be consistent with self-coupling requirements on a generally covariant Lagrangian. A consistency condition is given explicitely by Wald [28].
[42] This may seem to contradict Schwarzschild metric findings where the fall distance d approaches a constant value for $v \to c$. However, with increasing kinetic energy the influence of the probe on the metric cannot be neglected anymore. Within energy regimes accessible to current experiments the *NatAliE* prediction (40) is barely a correction. For example, in order to get $d = 1\,\mathrm{m}$ and $f = 1\,\mathrm{mm}$ you would need $E_{\mathrm{kin}}/E_{\mathrm{rest}} \approx 10^{13}$, far beyond the $\sim 4 \times 10^5$ reached for electrons in LEP at CERN. At $E_{\mathrm{kin}}/E_{\mathrm{rest}} \approx 10^{40}$ a proton would technically become a black hole.