Corrigendum to Dirac equation on hyperbolic octonions

Jens Köplinger - 1 Nov 2007

The multiplication table for conic sedenions used in [1] was not conforming to definitions in the articles cited therein: The classical circular octonion element "l" was not identified with "i_4", but instead mapped to "$-i_4$". To correct for this discrepancy, two defining relations need to be modified, and subsequent calculations adjusted. The essence of the article remains unchanged.

The particle's wave function Ψ from [1] relation (3) is correctly mapped to a hyperbolic octonion when changing the sign of the ψ_2^{r} component:

$$\Psi_{\mathrm{hyp8}} := \left(\psi_0^{\mathrm{r}}, \psi_0^{\mathrm{i}}, \psi_1^{\mathrm{r}}, \psi_1^{\mathrm{i}}, \quad \psi_2^{\mathrm{r}}, -\psi_2^{\mathrm{i}}, -\psi_3^{\mathrm{r}}, -\psi_3^{\mathrm{i}}\right)$$

The correct definition of the operator $\bigtriangledown_{\mathrm{hyp8}}$ from (4) has the opposite sign of the ∂_0 component:

$$\bigtriangledown_{\mathrm{hyp8}} := (-m, \partial_0, 0, 0, \quad 0, -\partial_3, \partial_2, -\partial_1)$$

All subsequent calculations can then be executed using these modified definitions, and yield correct results. No further changes in interpretation or wording are needed.

In detail, relations (14) and (15) become:

$$\begin{aligned}
&(-m, \partial_0, 0, 0, \quad 0, -\partial_3, \partial_2, -\partial_1)\left(\psi_0^{\mathrm{r}}, \psi_0^{\mathrm{i}}, \psi_1^{\mathrm{r}}, \psi_1^{\mathrm{i}}, \quad \psi_2^{\mathrm{r}}, -\psi_2^{\mathrm{i}}, -\psi_3^{\mathrm{r}}, -\psi_3^{\mathrm{i}}\right)\\
= \;& (-m\psi_0^{\mathrm{r}},\ -m\psi_0^{\mathrm{i}},\ -m\psi_1^{\mathrm{r}},\ -m\psi_1^{\mathrm{i}},\ -m\psi_2^{\mathrm{r}},\ m\psi_2^{\mathrm{i}},\ m\psi_3^{\mathrm{r}},\ m\psi_3^{\mathrm{i}})\\
&+(-\partial_0\psi_0^{\mathrm{i}},\ \partial_0\psi_0^{\mathrm{r}},\ -\partial_0\psi_1^{\mathrm{i}},\ \partial_0\psi_1^{\mathrm{r}},\ \partial_0\psi_2^{\mathrm{i}},\ \partial_0\psi_2^{\mathrm{r}},\ -\partial_0\psi_3^{\mathrm{i}},\ \partial_0\psi_3^{\mathrm{r}})\\
&+(\partial_3\psi_2^{\mathrm{i}},\ -\partial_3\psi_2^{\mathrm{r}},\ -\partial_3\psi_3^{\mathrm{i}},\ \partial_3\psi_3^{\mathrm{r}},\ -\partial_3\psi_0^{\mathrm{i}},\ -\partial_3\psi_0^{\mathrm{r}},\ -\partial_3\psi_1^{\mathrm{i}},\ \partial_3\psi_1^{\mathrm{r}})\\
&+(-\partial_2\psi_3^{\mathrm{r}},\ -\partial_2\psi_3^{\mathrm{i}},\ \partial_2\psi_2^{\mathrm{r}},\ \partial_2\psi_2^{\mathrm{i}},\ \partial_2\psi_1^{\mathrm{r}},\ -\partial_2\psi_1^{\mathrm{i}},\ \partial_2\psi_0^{\mathrm{r}},\ \partial_2\psi_0^{\mathrm{i}})\\
&+(\partial_1\psi_3^{\mathrm{i}},\ -\partial_1\psi_3^{\mathrm{r}},\ \partial_1\psi_2^{\mathrm{i}},\ -\partial_1\psi_2^{\mathrm{r}},\ -\partial_1\psi_1^{\mathrm{i}},\ -\partial_1\psi_1^{\mathrm{r}},\ \partial_1\psi_0^{\mathrm{i}},\ -\partial_1\psi_0^{\mathrm{r}})\\
= \;& (\ -m\psi_0^{\mathrm{r}} - \partial_0\psi_0^{\mathrm{i}} + \partial_3\psi_2^{\mathrm{i}} - \partial_2\psi_3^{\mathrm{r}} + \partial_1\psi_3^{\mathrm{i}}\ ,\\
&\ -m\psi_0^{\mathrm{i}} + \partial_0\psi_0^{\mathrm{r}} - \partial_3\psi_2^{\mathrm{r}} - \partial_2\psi_3^{\mathrm{i}} - \partial_1\psi_3^{\mathrm{r}}\ ,\\
&\ -m\psi_1^{\mathrm{r}} - \partial_0\psi_1^{\mathrm{i}} - \partial_3\psi_3^{\mathrm{i}} + \partial_2\psi_2^{\mathrm{r}} + \partial_1\psi_2^{\mathrm{i}}\ ,\\
&\ -m\psi_1^{\mathrm{i}} + \partial_0\psi_1^{\mathrm{r}} + \partial_3\psi_3^{\mathrm{r}} + \partial_2\psi_2^{\mathrm{i}} - \partial_1\psi_2^{\mathrm{r}}\ ,\\
&\ -m\psi_2^{\mathrm{r}} + \partial_0\psi_2^{\mathrm{i}} - \partial_3\psi_0^{\mathrm{i}} + \partial_2\psi_1^{\mathrm{r}} - \partial_1\psi_1^{\mathrm{i}}\ ,\\
&\ m\psi_2^{\mathrm{i}} + \partial_0\psi_2^{\mathrm{r}} - \partial_3\psi_0^{\mathrm{r}} - \partial_2\psi_1^{\mathrm{i}} - \partial_1\psi_1^{\mathrm{r}}\ ,\\
&\ m\psi_3^{\mathrm{r}} - \partial_0\psi_3^{\mathrm{i}} - \partial_3\psi_1^{\mathrm{i}} + \partial_2\psi_0^{\mathrm{r}} + \partial_1\psi_0^{\mathrm{i}}\ ,\\
&\ m\psi_3^{\mathrm{i}} + \partial_0\psi_3^{\mathrm{r}} + \partial_3\psi_1^{\mathrm{r}} + \partial_2\psi_0^{\mathrm{i}} - \partial_1\psi_0^{\mathrm{r}}\ )
\end{aligned}$$

Definition (19) becomes $\bigtriangledown_{\mathrm{Q1}} := (-m, \partial_0, 0, 0, 0, 0, 0, 0, \quad 0, 0, 0, 0, 0, 0, 0, 0)$ and definition (24) becomes

$$\Psi_{\mathrm{Q2}} := \left(0, 0, 0, 0, -\psi_2^{\mathrm{r}}, \psi_2^{\mathrm{i}}, \psi_3^{\mathrm{r}}, \psi_3^{\mathrm{i}}, \quad 0, 0, 0, 0, 0, 0, 0, 0\right)$$

Acknowledgements

My sincere thanks to Dr. Süleyman Demir, Anadolu University, for substantiating this correction.

References

[1] J. Köplinger, Dirac equation on hyperbolic octonions. *Appl. Math. Comput.* **182** (2006), 443-446.

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