

# Hypernumbers and relativity — reconstructing General Relativity from a projection ansatz

A research digest of Köpflinger (2007)

## Abstract

This is a guided tour of the paper **J. Köpflinger**, “**Hypernumbers and relativity**”, *Appl. Math. Comput.* **188** (2007) 954–969 (DOI: 10.1016/j.amc.2006.10.051). The paper does two things. First, it contains a standalone reconstruction of linearized General Relativity from a projection ansatz that takes gravity to be natively 4D-Euclidean and a Minkowski-native observer to see the projection of an inverse-square Euclidean force through a specific pair of mass-and-length rules. The paper’s Lemma 6 establishes equivalence with the linearized Einstein field equations for arbitrary mass distributions in arbitrary motion; the standard bootstrap of self-coupling carries that to the full covariant Einstein equations. Second, it embeds both the Minkowski metric of matter and the Euclidean 4-metric of gravity inside a single 16-dimensional complex-octonion ambient under a single invariance condition — one relativity principle for both sectors. This digest is organized in two parts: **Part I** treats the projection ansatz as a standalone reconstruction of linearized GR; **Part II** describes the algebraic embedding and what it contributes structurally.

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## 1 At a glance

This paper does two things, and with the benefit of nearly two decades of hindsight the second is the more useful one to lead with.

It defines a **single relativity principle** — invariance of a hypernumber modulus — on the 16-dimensional complex octonions, and shows how that condition specializes to Special Relativity on one sub-algebra and to a new Euclidean relativity on a second sub-algebra. That algebraic content is the natural continuation of the earlier papers in this series and is the explicit framing of the original.

But the paper also contains a **standalone reconstruction of linearized General Relativity from a projection ansatz**. The projection ansatz can be stated without any reference to octonions: postulate that gravity is natively 4D-Euclidean and obeys an inverse-square law on that space; postulate that a Minkowski-native observer sees the projection of that Euclidean force through a specific pair of mass-and-length transformation rules; carry the projection through; and the result is the linearized Einstein field equations of GR. The paper’s **Lemma 6** establishes the equivalence with linearized GR for arbitrary mass distributions in arbitrary motion, and the standard bootstrap of self-coupling carries linearized GR to the full covariant Einstein equations. With

the mass-projection correction identified in [KoeplAutotopies2023] in place — see §7 below — the equivalence is exact; the construction is empirically equivalent to GR in the domain where GR has itself been confirmed and differs from it in conceptual starting point, not in predictions.

Crucially, the projection ansatz itself does not require the metric-tensor formalism. The reconstruction works directly from an inverse-square law on Euclidean 4-space plus the mass-and-length projection rules. The metric-tensor language is brought in only at the end, to express the resulting field equation in the form Einstein’s GR uses, so that the two can be compared and the equivalence established (Appendix A).

This digest is organized in two parts. **Part I** treats the projection ansatz as a standalone reconstruction of linearized GR — the part of the paper that has aged best, and that is interesting in its own right regardless of where one stands on the algebraic ambient. **Part II** then describes the octonion embedding that the paper actually uses to motivate the two postulates from a single invariance condition, and points out the suggestive structural features of that ambient that earn it a closer look.

## 2 Who this digest is for

Readers familiar with Special and General Relativity at the level of a standard textbook — comfortable with metric tensors, the Lorentz transformation, and the weak-field / linearized form of Einstein’s field equations

$$\square \bar{h}_{\mu\nu} = -16\pi T_{\mu\nu} \quad (2.1)$$

(with  $c = \hbar = G = 1$ ). Part I is self-contained for such a reader. Part II builds on the split-octonion Dirac construction [Koepl2006] and on the  $\alpha$ -rotation [Koepl2007a], but is followable in outline without those.

## 3 Note on terminology

As in the earlier papers, the original uses the *Musean hypernumber* vocabulary:

Name used in the paper	Conventional modern name
conic sedenion	complex octonion
hyperbolic octonion	split-octonion
circular octonion	octonion

The two vocabularies are synonymous — the mathematics is the same algebra under either name. This digest uses the conventional names.

## 4 Note on mass and the equivalence principle

A point worth making early, before any formulas appear. Throughout this digest, the body’s **mass** means its **invariant rest mass**  $m_{\text{hyp}}$  — the same single quantity that appears in Newton’s second law and in the energy–momentum relation. We never introduce a second, distinct “gravitational mass” parameter. Whether the body is observed at rest or in relative motion,  $m_{\text{hyp}}$  is unchanged.

The projection ansatz of §7 below introduces an **effective gravitational source mass**  $m_{\text{cir}}$  that the moving observer sees as growing with the relative velocity  $|\mathbf{v}|$  between the two attracting

bodies. This is **not** a proposal that gravitational and inertial mass disagree as physical parameters — the **strong equivalence principle is not violated**. It is an *interpretive* statement: the inverse-square law in Euclidean 4-space, projected through the circular Lorentz transformation, *presents itself* to a Minkowski-native observer as a velocity-dependent enhancement of the gravitational coupling. The underlying invariant is the same  $m_{\text{hyp}}$  the observer would assign to the body at rest.

This stance has historical precedent worth flagging. In [Kraichnan1956], R. H. Kraichnan — the *father of spin-2 gravity*, whose linearized-graviton construction is a cornerstone of one of the canonical derivations of General Relativity — investigated under what circumstances a field theory of gravity could permit gravitational and inertial mass to differ. The present construction sits in similar interpretive territory, although on the other side of the same coin: rather than postulating a discrepancy at the level of the parameters, we observe that an apparent discrepancy in the *effective* gravitational mass arises naturally from a projection between two arithmetic backdrops, without splitting the rest mass into two physical parameters.

We note as a historical aside that Kraichnan, after his early work on gravitation, abandoned the line in the public record altogether and spent the rest of his career on fluid dynamics — a striking pivot for the founder of spin-2 gravity. The interpretive question of “what counts as the gravitational mass” recurs in the literature when a researcher steps outside the orthodox derivation of GR; we are stepping outside in a different way, but the same question has to be answered, and our answer is the one above.

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## Part I — Reconstructing General Relativity from a projection ansatz

### 5 Two metric spacetimes

There are two metric spacetimes of interest, both with coordinates  $(t, \mathbf{x})$  on  $\mathbb{R}^4$ :

1. **Minkowski spacetime.** The invariant  $ds^2 = dt^2 - |\mathbf{dx}|^2$  is preserved by the ordinary Lorentz transformation  $\Lambda$  for a pair of frames in constant relative motion  $\mathbf{v}$ . This is the home metric of matter, electromagnetism, and the Standard Model.
2. **Euclidean 4-space.** The invariant  $d\tilde{s}^2 = dt^2 + |\mathbf{dx}|^2$  is preserved by a **circular Lorentz transformation**  $\Lambda_{\text{cir}}$  — the same kind of “linear transformation that fixes the invariant of the space” construction, but adapted to the all-plus signature. The projection ansatz takes this Euclidean four-space as the home metric of gravity.

These are two different arithmetic backdrops for the same physical  $(t, \mathbf{x})$  coordinates. The ansatz is that gravity lives on the second, while everything else lives on the first.

### 6 The circular Lorentz transformation

Special Relativity’s Lorentz transformation is the linear map that fixes  $ds^2 = dt^2 - |\mathbf{dx}|^2$  between a pair of frames in constant relative motion. Build the analogue for Euclidean 4-space and one gets,

with the  $x_1$ -axis oriented along the line connecting any two attracting masses (equation (22) of the paper):

$$\Lambda_{\text{cir}} = \begin{pmatrix} (1 + |\mathbf{v}|^2)^{-1/2} & |\mathbf{v}| (1 + |\mathbf{v}|^2)^{-1/2} & 0 & 0 \\ -|\mathbf{v}| (1 + |\mathbf{v}|^2)^{-1/2} & (1 + |\mathbf{v}|^2)^{-1/2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (6.1)$$

Two features are worth calling out:

- **It fixes an all-plus modulus.** Whereas the Minkowski line element can be null or even negative, the Euclidean  $dt^2 + |d\mathbf{x}|^2$  is strictly positive. There is no light cone; past and future play the same role.
- **It is oriented toward a source mass.** The  $x_1$ -axis in the matrix is aligned along the vector connecting two attracting masses. In Special Relativity the Lorentz transformation is global — defined by the direction of the observer’s relative motion, not by the location of any other object. Here the transformation is **local to the gravitational interaction between a specific pair of bodies**. This is the detail where the ansatz departs from a literal symmetry principle and takes on its pairwise-gravitational-force character.

## 7 The projection ansatz: mass and length

For a point mass  $m$  moving at velocity  $\mathbf{v}$  relative to an observer, the projection ansatz consists of two rules, read directly off the circular Lorentz transformation:

- **Mass.** The “circular” (Euclidean-native) gravitational source mass, and the “hyperbolic” (Minkowski-native) invariant rest mass, relate via

$$m_{\text{cir}} = m_{\text{hyp}} \sqrt{1 + |\mathbf{v}|^2}. \quad (7.1)$$

For small  $|\mathbf{v}|$  this expands as  $m_{\text{hyp}}(1 + \frac{1}{2}|\mathbf{v}|^2 + \dots)$  — a velocity-dependent increase in the gravitational source mass, captured here as a direct consequence of the circular transformation, not as an independent postulate.

- **Length.** Let  $|\mathbf{x}|$  be the Minkowski observer’s coordinate distance to the source mass, and let  $|\mathbf{x}'|$  be the *effective gravitational distance* the inverse-square law in Euclidean 4-space operates on. Along the line of relative motion, these relate via

$$|\mathbf{x}| = |\mathbf{x}'| \sqrt{1 + |\mathbf{v}|^2}, \quad \text{equivalently} \quad |\mathbf{x}'| = \frac{|\mathbf{x}|}{\sqrt{1 + |\mathbf{v}|^2}}. \quad (7.2)$$

The Minkowski observer perceives the gravitational interaction as if it operated on a *contracted* effective distance  $|\mathbf{x}'|$ , smaller than its own coordinate distance  $|\mathbf{x}|$  by a factor of  $1/\sqrt{1 + |\mathbf{v}|^2}$ . This is the gravity-side counterpart of Lorentz length contraction in the hyperbolic sub-algebra: where moving rods in Special Relativity appear contracted, the gravitational coupling here appears to operate at a contracted apparent distance.

- **Perpendicular directions are unchanged**, just as in the hyperbolic case.

The mass-projection formula (7.1) is the **corrected** form of the paper’s eq. (56), which as published carried an additional factor  $(1 - |\mathbf{v}|^2)^{-1/2}$  in the denominator. That factor was identified as a derivation slip in footnote 7 of [KoeplAutotopies2023]: only the body’s *invariant* rest mass should be projected from Euclidean geometry, not its effective mass after a Minkowski Lorentz transformation. The corrected rule above is what the digest uses throughout. Appendix A.5 spells out the consequence: with this correction in place, the linearized Einstein field equation arrived at via the projection ansatz matches the standard form exactly, with no leftover factor.

**Static-potential enhancement.** The two rules combine in the inverse-square gravitational potential to produce a velocity- dependent **enhancement**, not a cancellation. Substituting the mass projection into the numerator and the contracted effective gravitational distance into the denominator,

$$\Phi(\mathbf{x}) = -\frac{m_{\text{cir}}}{|\mathbf{x}'|} = -\frac{m_{\text{hyp}} \sqrt{1 + |\mathbf{v}|^2}}{|\mathbf{x}|/\sqrt{1 + |\mathbf{v}|^2}} = -\frac{m_{\text{hyp}} (1 + |\mathbf{v}|^2)}{|\mathbf{x}|}. \quad (7.3)$$

Both projection factors land in the numerator: the mass rule contributes  $\sqrt{1 + |\mathbf{v}|^2}$  directly, and the contracted gravitational distance  $|\mathbf{x}'| = |\mathbf{x}|/\sqrt{1 + |\mathbf{v}|^2}$  contributes another  $\sqrt{1 + |\mathbf{v}|^2}$  via  $1/|\mathbf{x}'|$ . The static gravitational potential between two masses in relative motion is therefore  $(1 + |\mathbf{v}|^2)$  times the rest-frame Newtonian potential, evaluated at the Minkowski observer’s coordinate distance  $|\mathbf{x}|$ .

This  $(1 + |\mathbf{v}|^2)$  enhancement matches, at leading order in  $|\mathbf{v}|^2$ , the boost factor  $\gamma^2 = (1 - |\mathbf{v}|^2)^{-1} \approx 1 + |\mathbf{v}|^2 + \mathcal{O}(|\mathbf{v}|^4)$  that linearized General Relativity attaches to the energy density  $T_{00}$  of a moving source. It is the precursor of the velocity-dependent source-term structure that the field equation derivation below picks up; the formal proof that this matches the linearized Einstein form is in Appendix A.

The paper’s own name for this package of rules is the “Natural Alignment of Elementary Equations” (NatAliE) program of its Proposition 4; we will refer to it simply as the projection ansatz in what follows.

## 8 From projection rules to a field equation

Apply the projection rules of the previous section to a general mass distribution in arbitrary motion, with linear superposition over masses, and the result is the field equation

$$\square \bar{h}_{\mu\nu} = -16\pi \left( M_{\mu\nu} - \frac{1}{2} \rho \eta_{\mu\nu} \right), \quad (8.1)$$

where  $\bar{h}_{\mu\nu}$  is the metric perturbation in trace-reversed form,  $M_{\mu\nu}$  is the energy-momentum tensor of the mass distribution,  $\rho$  is the total invariant mass density, and  $\eta_{\mu\nu}$  is the Minkowski metric.

**This is the linearized Einstein field equation.**

A few things are worth calling out about the derivation:

- **No metric-tensor input on the gravitational side.** The projection ansatz operates entirely with scalar quantities — invariant masses, distances, the inverse-square gravitational law on Euclidean 4-space, and the projection rules of the previous section. The metric-tensor language enters at the *output* end, purely to express the result in the form Einstein’s GR uses, so the two formulations can be compared. **The projection ansatz itself does not use the geometric or covariant-derivative machinery of General Relativity.**

- **No general-covariance or equivalence-principle input.** The derivation proceeds by direct computation. The result happens to be Lorentz-covariant because the projection rules are constructed to be so (the underlying invariant-modulus condition forces it), but Lorentz covariance is an output, not an input.
- **Speed-of-light delay enters via the wave operator.** The retarded-potential structure of the linearized field equation arises because gravitational signals propagate at the speed of light — a postulate the projection ansatz inherits from the invariant-modulus structure of the hyperbolic limit (Special Relativity).

The full derivation — explicit projection-rule computations carried through to the linearized Einstein field equation — is given in **Appendix A**. The reader should be able to follow it from this digest without further reference to the original paper.

## 9 Equivalence with General Relativity

The paper’s **Lemma 6** states that the projection ansatz, applied to any mass density distribution in arbitrary motion, is equivalent to the linearized field equations of General Relativity. With the corrected mass projection of the previous section in place, the equivalence is exact: the linearized Einstein field equation arrived at through projection is the same as the one arrived at through the Einstein derivation.

Once the linearized field equation is in hand, the standard “bootstrap” argument [Deser1987, MTW] takes it to the full covariant nonlinear Einstein equations through self-interaction of the gravitational field. This argument is independent of how the linearized form was arrived at; it carries through here without modification.

The effect is that the projection-ansatz program is **empirically equivalent to General Relativity** in the domain where GR has itself been experimentally confirmed — same predictions, different conceptual starting point.

The detailed proof of equivalence — explicit derivation from the projection rules to the linearized Einstein field equation, including the technical step of repackaging the result in metric-tensor language — is given in **Appendix A**.

## 10 A conceptually different starting point

Writing down the linearized Einstein field equations from a projection rule is not the only way to reconstruct that object. Several routes exist in the literature:

- The **Einstein derivation**: postulate a dynamic geometry on spacetime, impose general covariance and the equivalence principle, and read off the field equations from variational first principles.
- The **spin-2 derivation**: postulate a massless spin-2 field on flat Minkowski spacetime and consistently self-couple it to its own energy-momentum tensor. Proven equivalent to the Einstein derivation — the spin-2 field dresses flat spacetime into an effective curved one through the bootstrap.

- The **projection derivation** (this paper): postulate that gravity is natively 4D-Euclidean, with its own inverse-square law and its own Lorentz-like invariance, and project the resulting force onto Minkowski spacetime where matter dynamics actually live.

None of these three is the unique “correct” way to build linearized GR — each answers a different conceptual question. The paper’s claim is not that its projection derivation is superior; it is that *a projection derivation is possible at all*, and that the result sits inside the same domain of empirical validity as the more familiar derivations.

**Why a projection, conceptually?** The motivation is anthropocentric and, consciously, philosophical rather than physical. Human beings and the instruments they build are made of atoms and molecules in electromagnetic interaction; electromagnetism is native to Minkowski spacetime; what we directly experience as “dynamics” is Lorentzian. Gravity, in contrast, is known to us mostly through one highly degenerate fact — that “everything falls down” — and is in that specific sense foreign to unaided human perception. The paper treats that asymmetry as a freedom: gravity can be taken to be native to a different (4D Euclidean) arithmetic, with its Minkowski-observer appearance the result of a projection. The pair of postulates becomes:

- **For matter and electromagnetism:** the speed of light is constant across non-accelerated frames, and dynamics live on Minkowski spacetime.
- **For gravity:** an inverse-square law is invariant across non-accelerated frames with respect to an Euclidean four-space, and the Minkowski-native observer sees the projection.

Both postulates carry over the equivalence of heavy and inert mass, and the equivalence of mass and energy, from standard physics.

The motivation is consciously soft — a conceptual preference, not an argument from empirical necessity. The empirical claim lives entirely on the equivalence with linearized GR established in the previous section.

## 11 An intuitive picture: frame-dragging and Lense–Thirring

An experimental consequence of the linearized field equations is the **Lense–Thirring effect** (and, more broadly, gravitomagnetic frame-dragging): a spinning massive body drags local inertial frames along with it, causing, for instance, the axis of a gyroscope in orbit to precess. The canonical GR derivation of the effect is not conceptually hard, but it requires working with the off-diagonal  $g_{0i}$  components of a stationary-rotating metric and reading off the gyroscope’s precession as parallel transport in that curved geometry. Visualizing what is going on from that starting point is genuinely difficult.

The projection picture offers an alternative mental image. The spinning source mass carries gravitational mass-currents — a rotating distribution of  $M_{\text{cir}}$  — which on Euclidean 4-space generate an inverse-square-like force with a direct analogue of a magnetic component, exactly as a moving electric charge generates a magnetic field. Project that force to the Minkowski frame where the gyroscope actually sits, and the gyroscope’s precession becomes a concrete torque on a concrete angular-momentum vector, with the direction predictable from the geometry of the source in much the same way one predicts the direction of the magnetic field of a current loop.

The paper itself does not compute Lense–Thirring explicitly — its Lemma 6 establishes the general equivalence once and covers the specific case by implication. An earlier-stage ResearchGate

preprint [Koepl2005Calc] worked out individual cases of the projection picture by hand. That 2005 document is an initial draft of the line of work — written before the paper here was developed — and is best read as the exploratory scratchpad from which the more robust later work sourced. The calculations sketched there include:

- **gravitational redshift** of light climbing out of a static mass distribution;
- the **event horizon of a black hole** (Schwarzschild-radius analogue) in the projection picture;
- **planetary perihelion shift** in the Schwarzschild-like field of a central mass;
- **deflection of a fast moving body or light** by a static central mass;
- a first attempt at **quantum gravitation** in the example of spin- $\frac{1}{2}$  particles, including a differential Coulomb-scattering cross section in the lowest order.

The point of Lemma 6 in the 2007 paper is that those individual cases are all covered, more cleanly, by a single general-case proof.

What remains useful about the projection picture is the intuitive one: **the projection gives a reader the ability to “see” where a given frame-dragging effect comes from**, in the same visual way electromagnetism affords, without having to run through a coordinate-system manipulation in curved spacetime.

## 12 What's still missing: from math tool to physics principle

A candid read of the paper's own outlook section is that the construction, as it stands, is a **valid mathematical description** of large-body gravity, but not yet a **physics principle** in the full sense.

What it lacks is conceptual simplicity. A physics principle — geodesic motion on curved spacetime, say, or the constancy of the speed of light — is a one-line statement about the world that then forces everything else. The projection ansatz, as presented, is a longer statement: *there are two arithmetics; gravity sits in one and matter in the other; and here are the rules to project one onto the other*. That is machinery, not a principle.

There are two honest ways this machinery could mature into a principle:

- A **quantum** route. The classical large-body projection here can be read as the non-quantum limit of a continuous one-parameter family — the “gravity phase” of [Koepl2023]. If that quantum extension is itself a sound physical theory, then the two arithmetics of Part I would be a consequence of pinning a single underlying parameter to its two distinguished classical values, not a postulate.
- A **foundational** route. If one could argue from first principles — from some operational or informational axiom — that matter interactions must be Minkowski-native and gravity must be Euclidean-native, the pair of postulates would reduce to one. Work in this direction is ongoing under the operational-reconstruction program, to which the present paper's projection construction is a structural input rather than a completed consequence.

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Neither is settled. The paper’s own position is modest: the projection is offered as a distinct tool in the kit, to be used and refined, not as a replacement for any existing account of gravity.

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## Part II — The octonion embedding

Part I treated the projection ansatz on its own terms — two metric spacetimes, a pair of transformation rules, a derivation of linearized GR. As a standalone reconstruction the ansatz is valuable: it adds a concrete, third route to the linearized field equations alongside the Einstein and spin-2 derivations, with a distinct physical picture and a distinct dataflow.

But standalone, the ansatz has a structural awkwardness. It carries **two independent invariance conditions** — Lorentz invariance for matter and circular-Lorentz invariance for gravity — postulated side by side. This is the part of the paper’s own framing that, sixteen years on, remains the most suggestive and the least exhausted: there is an algebraic ambient in which both conditions appear as the **same** invariance condition, restricted to two sub-algebras of one larger structure. That is what the original paper actually constructs, and Part II describes it.

### 13 What the algebraic ambient adds

The 16-dimensional complex octonions carry a single modulus  $|d\tau_{\text{con16}}|$ . The paper’s **Theorem 1** elevates invariance of this modulus to a single relativity principle:

Two lab frames  $A$  and  $A'$  are equivalent with respect to physical law if a linear transformation from  $x_\mu$  to  $x'_\mu$  leaves the modulus  $dT = |d\tau_{\text{con16}}|$  invariant.

There is one condition, not two. The hyperbolic and circular sub-algebras of the complex octonions are 8-dimensional composition algebras with signature  $(4, 4)$  and  $(8, 0)$  respectively, and the single modulus reduces, on each sub-algebra, to one of the two metrics of Part I:

- On the **hyperbolic (split-octonion) sub-algebra**, the modulus reduces to  $\sqrt{dt^2 - |d\mathbf{x}|^2}$  — the Minkowski line element. The class of linear transformations that preserve it is the ordinary Lorentz transformation. (This is the paper’s Lemma 2.)
- On the **circular (octonion) sub-algebra**, the modulus reduces to  $\sqrt{dt^2 + |d\mathbf{x}|^2}$  — the Euclidean 4-space line element of Part I — and the preserving transformation is  $\Lambda_{\text{cir}}$ .

The two postulates of Part I are now restrictions, to the two sub-algebras, of one underlying postulate. That is the structural advantage of the embedding.

### 14 Special Relativity from the hyperbolic limit

For the embedding to qualify as a *physics* claim — one that could in principle contradict or confirm established relativity — the single condition must reproduce Special Relativity exactly in the hyperbolic limit. That is exactly what §2 and §3 of the paper establish:

- **§2.** Define equivalent frames as those related by a linear transformation that leaves  $dT$  invariant.

- **§3, Lemma 2.** At phase  $\alpha = \pi/2$ , the 16-component  $dT$  reduces to  $\sqrt{dt^2 - |d\mathbf{x}|^2}$  and the preserving transformation is the Lorentz transformation.
- **§3, Lemma 3.** Fourier-transforming the 4-positions to 4-momenta and computing the hyperbolic-octonion modulus of the resulting momentum object reproduces the mass-shell  $m^2 = E^2 - |\mathbf{p}|^2$ .

Special Relativity is on the table as the hyperbolic limit of the ambient construction before any gravity content begins. This is what lets the Euclidean sector — which is a new claim — be read as a modification of one specific slice of an already-rigorous setup, rather than as an arbitrary second theory.

## 15 The $\alpha$ -rotation: continuity between the two sectors

The two sub-algebras are not isolated. The  $\alpha$ -rotation of [Koepl2007a] makes them two limits of a continuous one-parameter family inside the same 16-dimensional ambient. The phase  $\alpha$  acts not on a single coordinate but inside the multiplication table of the ambient algebra, rotating the distinguished non-real square root of +1 in the  $(1, i_0)$  plane; at  $\alpha = \pi/2$  one gets the hyperbolic sub-algebra, at  $\alpha = 0$  one gets the circular sub-algebra, and intermediate values describe a continuous interpolation between the two.

For the classical projection of Part I this  $\alpha$ -degree-of-freedom is locked at one of the two distinguished values — gravity is read from  $\alpha = 0$ , matter from  $\alpha = \pi/2$ . The  $\alpha$ -degree of freedom only matters when one moves *off* those classical values, at which point it ceases to be a coordinate substitution and becomes a genuinely new physical degree of freedom. That is the doorway the 2023 *Int. J. Theor. Phys.* paper [Koepl2023] steps through.

## 16 Why the embedding is suggestive

Three features of the algebraic ambient earn it more attention than a typical “useful coincidence”:

- **One invariance condition, not two.** The two postulates of Part I — Lorentz invariance for matter and circular-Lorentz invariance for gravity — are restrictions of a single condition on the ambient algebra to its two sub-algebras. This is closer to a principle than the bare projection ansatz can manage on its own.
- **A continuous interpolation, not a switch.** The two metrics are joined by an angle, not a binary choice. Both the Euclidean-native and Minkowski-native limits are pieces of one continuous structure inside the multiplication table of an ambient non-associative algebra — not two distinct theories bolted together.
- **A home for quantum extensions.** The  $\alpha$ -phase is a genuine physical degree of freedom when not pinned to its classical limits. The projection of Part I is the classical limit of a larger construction that keeps  $\alpha$  open for quantum use, and the follow-on bicomplex paper of 2023 uses exactly that opening to derive testable scattering predictions in Born approximation.

These are structural observations, not predictions; the projection of Part I would be empirically equivalent to linearized GR with or without the embedding. What the embedding contributes

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is *suggestiveness* — a reason to take the two-arithmetic ansatz seriously as more than ad-hoc bookkeeping, and a path along which that ansatz might one day mature into a proper physics principle.

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## 17 How to read the paper

The paper is long for the series (16 journal pages) but well partitioned. A reading order depending on what you want:

- **For Part I content (projection ansatz, equivalence with linearized GR)**, read §4.0 (the proof-plan introduction), Proposition 4, and §4.3 (Lemma 6 and its constructive proof). That is roughly four journal pages and contains the whole standalone reconstruction.
- **For Part II content (the algebraic ambient and its embedding of Special Relativity)**, read §1, §2 (up to Theorem 1), and §3 (Lemmas 2 and 3).
- **For the comparison with other approaches to non-traditional number systems**, see §5.3.

A pre-publication version of this material — longer, with more background and explicit worked examples — appears as [Koepl2005RG]. Section 3.2.4 there is a longer and more discursive account of the conceptual comparison with General Relativity given here in §10 (Part I).

## 18 About this digest

For preprints and personal versions of the author’s papers, see the author’s web page at [KoeplWWW]. The author’s ResearchGate profile is [ResearchGateJK].

## 19 How to cite this digest

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## A From the projection ansatz to the linearized Einstein equations

This appendix supplies the proof that the projection ansatz of §7, applied to a general mass distribution in arbitrary motion, produces the linearized Einstein field equation. The argument involves three inputs only: the inverse-square law of gravity in Euclidean 4-space, the mass and length projection rules of §7, and linear superposition. The metric-tensor language enters at the end, to express the result in the form Einstein’s GR uses.

The argument follows the paper’s §4.3, with the corrected mass projection of §7 in place throughout. §A.5 below is the only place where the comparison with the original (uncorrected) form matters; the rest of the appendix uses the corrected rule directly.

### A.1 Single mass at rest, observer at rest

For a point mass  $m$  at the spatial origin and an observer at rest in the same frame, the projection-ansatz prediction for the gravitational potential is the inverse-square law in Euclidean 4-space, with no projection factors (since  $\mathbf{v} = 0$ ):

$$\Phi(\mathbf{x}') = -\frac{m}{|\mathbf{x}'|}. \quad (\text{A.1})$$

Identifying  $h_{00} = -2\Phi$  as the  $(0, 0)$  component of the metric perturbation — the standard linearized-GR identification, valid for weak fields and slow motion — gives

$$h_{00}^{(0)}(t, \mathbf{x}') = \frac{2m}{|\mathbf{x}'|}. \quad (\text{A.2})$$

This is the standard linearized-GR rest-frame solution for a single point mass.

### A.2 Single mass at rest, observer in uniform motion

For an observer moving at velocity  $\mathbf{v}$  relative to the mass, the projection rules give

$$m_{\text{cir}} = m\sqrt{1 + |\mathbf{v}|^2}, \quad |\mathbf{x}'| = \frac{|\mathbf{x}|}{\sqrt{1 + |\mathbf{v}|^2}}, \quad (\text{A.3})$$

where  $|\mathbf{x}|$  is the Minkowski observer's coordinate distance and  $|\mathbf{x}'|$  is the contracted effective gravitational distance. The inverse-square law on Euclidean 4-space, operating on  $|\mathbf{x}'|$ , gives

$$\Phi(\mathbf{x}) = -\frac{m_{\text{cir}}}{|\mathbf{x}'|} = -\frac{m\sqrt{1 + |\mathbf{v}|^2}}{|\mathbf{x}|/\sqrt{1 + |\mathbf{v}|^2}} = -\frac{m(1 + |\mathbf{v}|^2)}{|\mathbf{x}|}. \quad (\text{A.4})$$

The static gravitational potential is enhanced by a factor of  $(1 + |\mathbf{v}|^2)$  compared to the rest-frame Newtonian form. The metric perturbation in the moving observer's frame is therefore

$$h_{00}(t, \mathbf{x}) = \frac{2m(1 + |\mathbf{v}|^2)}{|\mathbf{x}|}. \quad (\text{A.5})$$

This  $(1 + |\mathbf{v}|^2)$  matches, at leading order in  $|\mathbf{v}|^2$ , the boost factor  $\gamma^2 = (1 - |\mathbf{v}|^2)^{-1} \approx 1 + |\mathbf{v}|^2 + \mathcal{O}(|\mathbf{v}|^4)$  that linearized General Relativity attaches to the boosted energy density  $T_{00}$  of a moving source — i.e., it is exactly the velocity-dependent source factor required to match the standard linearized Einstein equation.

### A.3 Frame-covariant generalization

The expression for  $h_{00}$  above is computed in the moving observer's frame for a single static source. To extend to an arbitrary frame and arbitrary motion of source and observer, the metric perturbation must be promoted to a 4-tensor. The full set of  $h_{\mu\nu}$  components for a static point mass at rest in a frame where the observer moves at  $\mathbf{v}$  is

$$h_{\mu\nu}^{(0)}(t, \mathbf{x}) = \frac{2m(1 + |\mathbf{v}|^2)}{|\mathbf{x}|} \delta_{\mu\nu}, \quad (\text{A.6})$$

where  $\delta_{\mu\nu}$  is the Kronecker delta (identity on diagonal). The  $(1 + |\mathbf{v}|^2)$  factor is the leading-order piece of the boosted-source-tensor structure that the frame-covariant generalization absorbs. Extending via Lorentz transformation to an arbitrary frame — the technical step at the heart of the paper’s §4.3.4–4.3.5 — and trace-reversing,

$$\bar{h}_{\mu\nu} := h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu} \quad \text{with} \quad h := h^\mu{}_\mu, \quad (\text{A.7})$$

the result for a single mass  $m_l$  in arbitrary motion is

$$\bar{h}_{\mu\nu}^{(l)} = 16\pi \left( M_{\mu\nu}^{(l)} - \frac{1}{2} m_l \eta_{\mu\nu} \right), \quad (\text{A.8})$$

where  $M_{\mu\nu}^{(l)}$  is the energy-momentum tensor of the single mass and  $\eta_{\mu\nu}$  is the Minkowski metric.

(This step is where the metric-tensor language is brought in. The projection ansatz produced  $h_{00}$  directly from the inverse-square law in §A.1–A.2; the tensor structure here is the repackaging of that scalar result into a Lorentz-covariant form, so the comparison with linearized GR can be made.)

## A.4 Generalization to arbitrary mass distributions

Linearity of the field equation allows superposition over masses. Defining

$$\rho := \sum_l m_l, \quad M_{\mu\nu} := \sum_l M_{\mu\nu}^{(l)}, \quad (\text{A.9})$$

the combined result is

$$\square \bar{h}_{\mu\nu} = -16\pi \left( M_{\mu\nu} - \frac{1}{2} \rho \eta_{\mu\nu} \right). \quad (\text{A.10})$$

This is exactly the linearized Einstein field equation in trace-reversed form.

## A.5 Where the original paper’s velocity factor came from

The original paper’s eq. (56) wrote the mass projection as

$$m_{\text{cir}(l)} = m_{\text{hyp}(l)} \sqrt{\frac{1 + |\mathbf{v}|^2}{(1 - |\mathbf{v}|^2)^2}}. \quad (\text{A.11})$$

The extra factor  $(1 - |\mathbf{v}|^2)^{-1}$  inside the square root is  $\gamma^2$ , with  $\gamma = (1 - |\mathbf{v}|^2)^{-1/2}$  the ordinary Lorentz factor. Carried through the derivation, that factor leaves a single residual  $\gamma$  multiplying the right-hand side of the linearized field equation, and the paper’s eq. (69) reads

$$\square \bar{h}_{\mu\nu} = -16\pi \gamma \left( M_{\mu\nu} - \frac{1}{2} \rho \eta_{\mu\nu} \right). \quad (\text{A.12})$$

The paper handles the discrepancy in words by appealing to the low-energy-density approximation, where  $\gamma \approx 1$ .

Footnote 7 of [KoeplAutopies2023] identifies the extra factor as a derivation slip: only the body’s *invariant* rest mass should have been projected from Euclidean geometry, not its *effective* mass after a Minkowski Lorentz transformation. With the correction in place — the projection

rule of §7 of this digest — the spurious  $(1 - |\mathbf{v}|^2)^{-1/2}$  disappears from eq. (56) onward, and the linearized field equation reads, at all velocities,

$$\square \bar{h}_{\mu\nu} = -16\pi(M_{\mu\nu} - \frac{1}{2}\rho\eta_{\mu\nu}), \quad (\text{A.13})$$

the standard linearized Einstein form, with no leftover factor and no need for a low-energy-density qualifier.

## A.6 From linearized to full nonlinear Einstein

Once the linearized field equation is in hand, the standard “bootstrap” argument [Deser1987, MTW] takes it to the full nonlinear Einstein equations through iterative self-coupling of the gravitational field via its own energy-momentum tensor. The argument is independent of how the linearized form was arrived at; it applies here without modification.

The projection-ansatz reconstruction is therefore equivalent to General Relativity in its domain of empirical validity.

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