Hypernumbers and relativity — reconstructing General Relativity from a projection ansatz

A research digest of Köplinger (2007)

Abstract

This is a guided tour of the paper **J. Köplinger, "Hypernumbers and relativity", *Appl. Math. Comput.* 188 (2007) 954–969** (DOI: 10.1016/j.amc.2006.10.051). The paper does two things. First, it contains a standalone reconstruction of linearized General Relativity from a projection ansatz that takes gravity to be natively 4D-Euclidean and a Minkowski-native observer to see the projection of an inverse-square Euclidean force through a specific pair of mass-and-length rules. The paper's Lemma 6 establishes equivalence with the linearized Einstein field equations for arbitrary mass distributions in arbitrary motion; the standard bootstrap of self-coupling carries that to the full covariant Einstein equations. Second, it embeds both the Minkowski metric of matter and the Euclidean 4-metric of gravity inside a single 16-dimensional complex-octonion ambient under a single invariance condition — one relativity principle for both sectors. This digest is organized in two parts: **Part I** treats the projection ansatz as a standalone reconstruction of linearized GR; **Part II** describes the algebraic embedding and what it contributes structurally.

Digest author: Jens Köplinger, 2026. *Produced*: 20 April 2026. This digest was generated with the help of AI and has been reviewed and edited by the author for accuracy and relevance.

1 At a glance

This paper does two things, and with the benefit of nearly two decades of hindsight the second is the more useful one to lead with.

It defines a **single relativity principle** — invariance of a hypernumber modulus — on the 16-dimensional complex octonions, and shows how that condition specializes to Special Relativity on one sub-algebra and to a new Euclidean relativity on a second sub-algebra. That algebraic content is the natural continuation of the earlier papers in this series and is the explicit framing of the original.

But the paper also contains a **standalone reconstruction of linearized General Relativity from a projection ansatz**. The projection ansatz can be stated without any reference to octonions: postulate that gravity is natively 4D-Euclidean and obeys an inverse-square law on that space; postulate that a Minkowski-native observer sees the projection of that Euclidean force through a specific pair of mass-and-length transformation rules; carry the projection through; and the result is the linearized Einstein field equations of GR. The paper's **Lemma 6** establishes the equivalence with linearized GR for arbitrary mass distributions in arbitrary motion, and the standard bootstrap of self-coupling carries linearized GR to the full covariant Einstein equations. The construction is empirically equivalent to GR in the domain where GR has itself been confirmed; it differs from GR in conceptual starting point, not in predictions.

This digest is organized in two parts. **Part I** treats the projection ansatz as a standalone reconstruction of linearized GR — the part of the paper that has aged best, and that is interesting in its own right regardless of where one stands on the algebraic ambient. **Part II** then describes the octonion embedding that the paper actually uses to motivate the two postulates from a single invariance condition, and points out the suggestive structural features of that ambient that earn it a closer look.

2 Who this digest is for

Readers familiar with Special and General Relativity at the level of a standard textbook — comfortable with metric tensors, the Lorentz transformation, and the weak-field / linearized form of Einstein's field equations

$$\Box \bar{h}_{\mu\nu} \;=\; -16\pi\, T_{\mu\nu} \tag{2.1}$$

(with $c = \hbar = G = 1$). Part I is self-contained for such a reader. Part II builds on the split-octonion Dirac construction [Koepl2006] and on the α-rotation [Koepl2007a], but is followable in outline without those.

3 Note on terminology

As in the earlier papers, the original uses the *Musean hypernumber* vocabulary:

Name used in the paper	Conventional modern name
conic sedenion	complex octonion
hyperbolic octonion	split-octonion
circular octonion	octonion

The two vocabularies are synonymous — the mathematics is the same algebra under either name. This digest uses the conventional names.

Part I — Reconstructing General Relativity from a projection ansatz

4 Two metric spacetimes

There are two metric spacetimes of interest, both with coordinates $(t, \mathbf{x})$ on $\mathbb{R}^4$:

1. **Minkowski spacetime.** The invariant $\mathrm{d}s^2 = \mathrm{d}t^2 - |\mathrm{d}\mathbf{x}|^2$ is preserved by the ordinary Lorentz transformation Λ for a pair of frames in constant relative motion $\mathbf{v}$. This is the home metric of matter, electromagnetism, and the Standard Model.

2. **Euclidean 4-space.** The invariant $\mathrm{d}\tilde{s}^2 = \mathrm{d}t^2 + |\mathrm{d}\mathbf{x}|^2$ is preserved by a **circular Lorentz transformation** Λ_{cir} — the same kind of "linear transformation that fixes the invariant of the space" construction, but adapted to the all-plus signature. The projection ansatz takes this Euclidean four-space as the home metric of gravity.

These are two different arithmetic backdrops for the same physical $(t, \mathbf{x})$ coordinates. The ansatz is that gravity lives on the second, while everything else lives on the first.

5 The circular Lorentz transformation

Special Relativity's Lorentz transformation is the linear map that fixes $\mathrm{d}s^2 = \mathrm{d}t^2 - |\mathrm{d}\mathbf{x}|^2$ between a pair of frames in constant relative motion. Build the analogue for Euclidean 4-space and one gets, with the x_1-axis oriented along the line connecting any two attracting masses (equation (22) of the paper):

$$\Lambda_{\mathrm{cir}} = \begin{pmatrix} (1+|\mathbf{v}|^2)^{-1/2} & |\mathbf{v}|\,(1+|\mathbf{v}|^2)^{-1/2} & 0 & 0 \\ -|\mathbf{v}|\,(1+|\mathbf{v}|^2)^{-1/2} & (1+|\mathbf{v}|^2)^{-1/2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{5.1}$$

Two features are worth calling out:

- **It fixes an all-plus modulus.** Whereas the Minkowski line element can be null or even negative, the Euclidean $\mathrm{d}t^2 + |\mathrm{d}\mathbf{x}|^2$ is strictly positive. There is no light cone; past and future play the same role.
- **It is oriented toward a source mass.** The x_1-axis in the matrix is aligned along the vector connecting two attracting masses. In Special Relativity the Lorentz transformation is global — defined by the direction of the observer's relative motion, not by the location of any other object. Here the transformation is **local to the gravitational interaction between a specific pair of bodies**. This is the detail where the ansatz departs from a literal symmetry principle and takes on its pairwise-gravitational-force character.

6 The projection ansatz: mass and length

For a point mass m moving at velocity $\mathbf{v}$ relative to an observer, the projection ansatz consists of two rules, read directly off the circular Lorentz transformation:

- **Mass.** The "circular" (Euclidean-native) mass that generates gravity, and the "hyperbolic" (Minkowski-native) inertial mass, relate via

$$m_{\mathrm{cir}} \;=\; m_{\mathrm{hyp}} \sqrt{\frac{1+|\mathbf{v}|^2}{(1-|\mathbf{v}|^2)^2}}. \tag{6.1}$$

  For small $|\mathbf{v}|$ this expands as $m_{\mathrm{hyp}}(1 + \frac{3}{2}|\mathbf{v}|^2 + \ldots)$ — the usual relativistic increase in gravitational mass, captured here as a direct consequence of the circular transformation, not as an independent postulate.

- **Length.** A distance $|\mathbf{x}|$ between two masses, as seen by the gravitational force on Euclidean 4-space, appears length-*expanded* relative to the same distance as measured on Minkowski spacetime along the line of relative motion:

$$|\mathbf{x}| \;=\; |\mathbf{x}'|\,\sqrt{1+|\mathbf{v}|^2}. \tag{6.2}$$

  This is the direct counterpart of Lorentz length contraction, run the other way. Distances *for the purposes of the gravitational force* are not Lorentz-contracted; they are expanded. The expansion is the Euclidean rotation's counterpart to the hyperbolic rotation's contraction.

- **Perpendicular directions are unchanged**, just as in the hyperbolic case.

The paper's own name for the package of rules is the "Natural Alignment of Elementary Equations" (NatAliE) program of its Proposition 4; we will refer to it simply as the projection ansatz in what follows.

7 From projection rules to linearized GR

Put the mass and length rules into the standard post-Newtonian gravitational equation of motion — the "Newton gravity generalized for Special Relativity" construction of the paper's §4.2.4, which on its own is a well-understood weak-field picture with no novel assumptions. Carry through the projections explicitly for a single point mass; superpose linearly for a general mass density distribution in arbitrary motion. The result is

$$\Box\,\overline{h}_{\mu\nu} \;=\; -16\pi\big(M_{\mu\nu} - \tfrac{1}{2}\rho\,\eta_{\mu\nu}\big)\,. \tag{7.1}$$

This is the **linearized Einstein field equation**.

The derivation is by direct computation. No appeal to general covariance, the equivalence principle, or a variational principle is made anywhere along the way; the field equation drops out as the linearized form of an ordinary inverse-square force on Euclidean 4-space, projected back to a Minkowski observer.

> **Note on a correction.** The published paper carries an additional factor $\gamma = (1-|\mathbf{v}|^2)^{-1/2}$ in front of the source term — its equation (69) — traceable to a stray $\sqrt{1-|\mathbf{v}|^2}$ that was introduced in the denominator at equation (56). The author has since identified that factor as an error of derivation: the projection from Euclidean geometry should act on the body's *invariant* mass, not on its *effective* mass after a Lorentz transformation. With that correction, the spurious $\sqrt{1-|\mathbf{v}|^2}$ is absent from (56) onward, the γ does not appear in (69), and the projection ansatz reproduces the linearized Einstein field equation exactly, as written above. See footnote 7 of [KoeplAutotopies2023] for this correction.

8 Equivalence with linearized General Relativity

Lemma 6 of the paper closes the loop. Paraphrasing slightly to match the language used here, the lemma states that the projection ansatz, applied to any mass density distribution in arbitrary motion, is equivalent to the linearized field equations from General Relativity. With the correction noted in the previous section, the equivalence is exact — no leftover factor, no higher-order remainder; the projection ansatz delivers the linearized Einstein field equation as written.

Once the linearized field equations are in hand, the standard bootstrap argument [Deser1987, MTW] takes them to the full covariant Einstein equations through self-interaction of the gravitational field. The paper cites this as a known result and does not re-derive it. The effect is that the projection-ansatz program, followed through, is empirically equivalent to General Relativity in the domain where GR has itself been experimentally confirmed.

9 A conceptually different starting point

Writing down the linearized Einstein field equations from a projection rule is not the only way to reconstruct that object. Several routes exist in the literature:

- The **Einstein derivation**: postulate a dynamic geometry on spacetime, impose general covariance and the equivalence principle, and read off the field equations from variational first principles.
- The **spin-2 derivation**: postulate a massless spin-2 field on flat Minkowski spacetime and consistently self-couple it to its own energy-momentum tensor. Proven equivalent to the Einstein derivation — the spin-2 field dresses flat spacetime into an effective curved one through the bootstrap.
- The **projection derivation** (this paper): postulate that gravity is natively 4D-Euclidean, with its own inverse-square law and its own Lorentz-like invariance, and project the resulting force onto Minkowski spacetime where matter dynamics actually live.

None of these three is the unique "correct" way to build linearized GR — each answers a different conceptual question. The paper's claim is not that its projection derivation is superior; it is that *a projection derivation is possible at all*, and that the result sits inside the same domain of empirical validity as the more familiar derivations.

Why a projection, conceptually? The motivation is anthropocentric and, consciously, philosophical rather than physical. Human beings and the instruments they build are made of atoms and molecules in electromagnetic interaction; electromagnetism is native to Minkowski spacetime; what we directly experience as "dynamics" is Lorentzian. Gravity, in contrast, is known to us mostly through one highly degenerate fact — that "everything falls down" — and is in that specific sense foreign to unaided human perception. The paper treats that asymmetry as a freedom: gravity can be taken to be native to a different (4D Euclidean) arithmetic, with its Minkowski-observer appearance the result of a projection. The pair of postulates becomes:

- **For matter and electromagnetism:** the speed of light is constant across non-accelerated frames, and dynamics live on Minkowski spacetime.
- **For gravity:** an inverse-square law is invariant across non-accelerated frames with respect to an Euclidean four-space, and the Minkowski-native observer sees the projection.

Both postulates carry over the equivalence of heavy and inert mass, and the equivalence of mass and energy, from standard physics.

The motivation is consciously soft — a conceptual preference, not an argument from empirical necessity. The empirical claim lives entirely on the equivalence with linearized GR established in the previous section.

10 An intuitive picture: frame-dragging and Lense–Thirring

An experimental consequence of the linearized field equations is the **Lense–Thirring effect** (and, more broadly, gravitomagnetic frame-dragging): a spinning massive body drags local inertial frames along with it, causing, for instance, the axis of a gyroscope in orbit to precess. The canonical GR derivation of the effect is not conceptually hard, but it requires working with the off-diagonal g_{0i} components of a stationary-rotating metric and reading off the gyroscope's precession as parallel transport in that curved geometry. Visualizing what is going on from that starting point is genuinely difficult.

The projection picture offers an alternative mental image. The spinning source mass carries gravitational mass-currents — a rotating distribution of M_{cir} — which on Euclidean 4-space generate an inverse-square-like force with a direct analogue of a magnetic component, exactly as a moving electric charge generates a magnetic field. Project that force to the Minkowski frame where the gyroscope actually sits, and the gyroscope's precession becomes a concrete torque on a concrete angular-momentum vector, with the direction predictable from the geometry of the source in much the same way one predicts the direction of the magnetic field of a current loop.

The paper does not compute Lense–Thirring explicitly — its Lemma 6 establishes the general equivalence once and covers the specific case by implication. The author has carried out individual cases (perihelion precession, light deflection) by hand in earlier unpublished work; the point of Lemma 6 is that those are all covered by the general-case proof. What remains useful about the picture is the intuitive one: **the projection gives a reader the ability to "see" where a given frame-dragging effect comes from**, in the same visual way electromagnetism affords, without having to run through a coordinate-system manipulation in curved spacetime.

11 What's still missing: from math tool to physics principle

A candid read of the paper's own outlook section is that the construction, as it stands, is a **valid mathematical description** of large-body gravity, but not yet a **physics principle** in the full sense.

What it lacks is conceptual simplicity. A physics principle — geodesic motion on curved spacetime, say, or the constancy of the speed of light — is a one-line statement about the world that then forces everything else. The projection ansatz, as presented, is a longer statement: *there are two arithmetics; gravity sits in one and matter in the other; and here are the rules to project one onto the other*. That is machinery, not a principle.

There are two honest ways this machinery could mature into a principle:

- A **quantum** route. The classical large-body projection here can be read as the non-quantum limit of a continuous one-parameter family — the "gravity phase" of [Koepl2023]. If that quantum extension is itself a sound physical theory, then the two arithmetics of Part I would be a consequence of pinning a single underlying parameter to its two distinguished classical values, not a postulate.

- A **foundational** route. If one could argue from first principles — from some operational or informational axiom — that matter interactions must be Minkowski-native and gravity must be Euclidean-native, the pair of postulates would reduce to one. Work in this direction is ongoing under the operational-reconstruction program, to which the present paper's projection construction is a structural input rather than a completed consequence.

Neither is settled. The paper's own position is modest: the projection is offered as a distinct tool in the kit, to be used and refined, not as a replacement for any existing account of gravity.

Part II — The octonion embedding

Part I treated the projection ansatz on its own terms — two metric spacetimes, a pair of transformation rules, a derivation of linearized GR. As a standalone reconstruction the ansatz is valuable: it adds a concrete, third route to the linearized field equations alongside the Einstein and spin-2 derivations, with a distinct physical picture and a distinct dataflow.

But standalone, the ansatz has a structural awkwardness. It carries **two independent invariance conditions** — Lorentz invariance for matter and circular-Lorentz invariance for gravity — postulated side by side. This is the part of the paper's own framing that, sixteen years on, remains the most suggestive and the least exhausted: there is an algebraic ambient in which both conditions appear as the **same** invariance condition, restricted to two sub-algebras of one larger structure. That is what the original paper actually constructs, and Part II describes it.

12 What the algebraic ambient adds

The 16-dimensional complex octonions carry a single modulus $|\mathrm{d}\tau_{\mathrm{con16}}|$. The paper's **Theorem 1** elevates invariance of this modulus to a single relativity principle:

> Two lab frames A and A' are equivalent with respect to physical law if a linear transformation from x_μ to x'_μ leaves the modulus $\mathrm{d}T = |\mathrm{d}\tau_{\mathrm{con16}}|$ invariant.

There is one condition, not two. The hyperbolic and circular sub-algebras of the complex octonions are 8-dimensional composition algebras with signature $(4,4)$ and $(8,0)$ respectively, and the single modulus reduces, on each sub-algebra, to one of the two metrics of Part I:

- On the **hyperbolic (split-octonion) sub-algebra**, the modulus reduces to $\sqrt{\mathrm{d}t^2 - |\mathrm{d}\mathbf{x}|^2}$ — the Minkowski line element. The class of linear transformations that preserve it is the ordinary Lorentz transformation. (This is the paper's Lemma 2.)
- On the **circular (octonion) sub-algebra**, the modulus reduces to $\sqrt{\mathrm{d}t^2 + |\mathrm{d}\mathbf{x}|^2}$ — the Euclidean 4-space line element of Part I — and the preserving transformation is Λ_{cir}.

The two postulates of Part I are now restrictions, to the two sub-algebras, of one underlying postulate. That is the structural advantage of the embedding.

13 Special Relativity from the hyperbolic limit

For the embedding to qualify as a *physics* claim — one that could in principle contradict or confirm established relativity — the single condition must reproduce Special Relativity exactly in the hyperbolic limit. That is exactly what §2 and §3 of the paper establish:

- **§2.** Define equivalent frames as those related by a linear transformation that leaves $\mathrm{d}T$ invariant.

- **§3, Lemma 2.** At phase $\alpha = \pi/2$, the 16-component $\mathrm{d}T$ reduces to $\sqrt{\mathrm{d}t^2 - |\mathrm{d}\mathbf{x}|^2}$ and the preserving transformation is the Lorentz transformation.
- **§3, Lemma 3.** Fourier-transforming the 4-positions to 4-momenta and computing the hyperbolic-octonion modulus of the resulting momentum object reproduces the mass-shell $m^2 = E^2 - |\mathbf{p}|^2$.

Special Relativity is on the table as the hyperbolic limit of the ambient construction before any gravity content begins. This is what lets the Euclidean sector — which is a new claim — be read as a modification of one specific slice of an already-rigorous setup, rather than as an arbitrary second theory.

14 The α-rotation: continuity between the two sectors

The two sub-algebras are not isolated. The α-rotation of [Koepl2007a] makes them two limits of a continuous one-parameter family inside the same 16-dimensional ambient. The phase α acts not on a single coordinate but inside the multiplication table of the ambient algebra, rotating the distinguished non-real square root of $+1$ in the $(1, i_0)$ plane; at $\alpha = \pi/2$ one gets the hyperbolic sub-algebra, at $\alpha = 0$ one gets the circular sub-algebra, and intermediate values describe a continuous interpolation between the two.

For the classical projection of Part I this α-degree-of-freedom is locked at one of the two distinguished values — gravity is read from $\alpha = 0$, matter from $\alpha = \pi/2$. The α-degree of freedom only matters when one moves *off* those classical values, at which point it ceases to be a coordinate substitution and becomes a genuinely new physical degree of freedom. That is the doorway the 2023 *Int. J. Theor. Phys.* paper [Koepl2023] steps through.

15 Why the embedding is suggestive

Three features of the algebraic ambient earn it more attention than a typical "useful coincidence":

- **One invariance condition, not two.** The two postulates of Part I — Lorentz invariance for matter and circular-Lorentz invariance for gravity — are restrictions of a single condition on the ambient algebra to its two sub-algebras. This is closer to a principle than the bare projection ansatz can manage on its own.
- **A continuous interpolation, not a switch.** The two metrics are joined by an angle, not a binary choice. Both the Euclidean-native and Minkowski-native limits are pieces of one continuous structure inside the multiplication table of an ambient non-associative algebra — not two distinct theories bolted together.
- **A home for quantum extensions.** The α-phase is a genuine physical degree of freedom when not pinned to its classical limits. The projection of Part I is the classical limit of a larger construction that keeps α open for quantum use, and the follow-on bicomplex paper of 2023 uses exactly that opening to derive testable scattering predictions in Born approximation.

These are structural observations, not predictions; the projection of Part I would be empirically equivalent to linearized GR with or without the embedding. What the embedding contributes is *suggestiveness* — a reason to take the two-arithmetic ansatz seriously as more than ad-hoc bookkeeping, and a path along which that ansatz might one day mature into a proper physics principle.

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16 How to read the paper

The paper is long for the series (16 journal pages) but well partitioned. A reading order depending on what you want:

- **For Part I content (projection ansatz, equivalence with linearized GR)**, read §4.0 (the proof-plan introduction), Proposition 4, and §4.3 (Lemma 6 and its constructive proof). That is roughly four journal pages and contains the whole standalone reconstruction.
- **For Part II content (the algebraic ambient and its embedding of Special Relativity)**, read §1, §2 (up to Theorem 1), and §3 (Lemmas 2 and 3).
- **For the comparison with other approaches to non-traditional number systems**, see §5.3.

A pre-publication version of this material — longer, with more background and explicit worked examples — appears as [Koepl2005RG]. Section 3.2.4 there is a longer and more discursive account of the conceptual comparison with General Relativity given here in §9 (Part I).

17 About this digest

For preprints and personal versions of the author's papers, see the author's web page at [KoeplWWW]. The author's ResearchGate profile is [ResearchGateJK].

18 How to cite this digest

J. Köplinger, "*Hypernumbers and relativity* — reconstructing General Relativity from a projection ansatz", research digest (2026). Available via ResearchGate.

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