Gravity and electromagnetism on conic sedenions — a complex octonion phase α as particle property

A research digest of Köplinger (2007)

Abstract

This is a guided tour of the paper **J. Köplinger, "Gravity and electromagnetism on conic sedenions", *Appl. Math. Comput.* 188 (2007) 948–953** (DOI: 10.1016/j.amc.2006.10.050). The paper concludes the four-paper proposal of 2006–2007, in the complex-octonion setting that was worked toward throughout the series. It adds an electromagnetic gauge potential A_μ to the complex-octonion Dirac construction of the earlier papers and shows two things: the same A_μ embedded in the split-octonion sub-algebra reproduces the standard electromagnetism-coupled Dirac equation, while embedded in the octonion sub-algebra it generates a candidate quantum-gravitational interaction; and a real mixing angle α between the two embeddings is forced by the relativity-of-modulus principle to be a constant of the particle, joining mass m and electric charge e as a third real parameter. For Standard Model particles α is pinned near $\pi/2$ with a tiny admixture $\beta^2 \sim m^2/e^2$, leaving the line element indistinguishable from Special Relativity at any energy currently accessible. The mixing angle α is one possible algebraic origin of the "gravity phase" that re-emerges sixteen years later as a phenomenological handle for quantum gravity in [Koepl2023]; that 2023 paper stands independently and supports the present construction together with several other possible origins it cites in turn.

Digest author: Jens Köplinger, 2026. *Produced*: 2 May 2026. This digest was generated with the help of AI and has been reviewed and edited by the author for accuracy and relevance.

1 At a glance

This is the paper that **concludes the four-paper proposal of 2006–2007**, in the complex-octonion setting that was worked toward throughout. Where the earlier papers built the algebraic backdrop — split-octonion Dirac equation [Koepl2006], octonion counterpart with gravity-like hallmarks [Koepl2007a], projection-ansatz reconstruction of linearized General Relativity [Koepl2007c] — this paper closes the construction by adding an **electromagnetic gauge potential** A_μ to the picture.

Two results matter today.

1. **One field, two forces.** The same gauge potential A_μ, when embedded into the split-octonion sub-algebra of the complex octonions, reproduces the standard electromagnetism-coupled Dirac equation $[\gamma^\mu(\hat{p}_\mu - eA_\mu) - m]\Psi = 0$ of physics. Embedded into the octonion sub-algebra under exactly the same algebraic rules, the same A_μ plays the role of a generator

of the quantum gravity candidate of [Koepl2007a]. The two embeddings differ only by the choice of sub-algebra; the field is one and the same.

2. **A third particle property.** Mixing the two embeddings on the ambient complex octonion takes a real angle α. At $\alpha = \pi/2$ the construction is pure electromagnetism on Minkowski; at $\alpha = 0$ it is the gravity candidate on Euclidean 4-space; in between, the two are mixed. The complex-octonion modulus is *not* invariant under a change of α, so the relativity-of-modulus principle of [Koepl2007c] forces α to be a constant of the particle, alongside its mass m and electric charge e. A particle in this construction carries the triple (m, e, α).

For Standard Model particles, α is pinned near $\pi/2$, with a small admixture $\beta = \pi/2 - \alpha$ whose square sets the gravitational-to-electromagnetic force ratio: $\beta^2 \sim m^2/e^2$ in natural units. For two electrons, $\beta_e^2 \sim 10^{-42}$. The corresponding correction to the spacetime line element is too small to detect at any energy currently accessible. Genuine complex-octonion arithmetic — where the line element cannot be written as a second-order tensor — is only required at extreme energies, the Big Bang regime.

The mixing angle α introduced here is *one possible* algebraic origin of the **gravity phase** that re-emerges sixteen years later in [Koepl2023] as a phenomenological handle for quantum gravity in scattering experiments. The 2023 paper stands on its own and provides phenomenological support for the present construction *alongside* several other possible origins it cites in turn.

A final framing note. Other possible avenues for combining gravity and electromagnetism on hypernumber arithmetic certainly exist; the construction laid out here is merely one suggested possibility.

2 Who this digest is for

Readers familiar with the electromagnetism-coupled Dirac equation, metric tensors, and the Lorentz factor $\gamma = (1 - |\mathbf{v}|^2)^{-1/2}$ of Special Relativity. The digest builds on the split-octonion Dirac construction [Koepl2006], the α-rotation of [Koepl2007a], and the projection ansatz of [Koepl2007c], but the physics content here is followable on its own.

3 Note on terminology

As in the earlier papers, the original uses the *Musean hypernumber* vocabulary:

Name used in the paper	Conventional modern name
conic sedenion	complex octonion
hyperbolic octonion	split-octonion
circular octonion	octonion

The two vocabularies are synonymous — the mathematics is the same algebra under either name. This digest uses the conventional names.

4 The setup: Dirac with EM field, on the split-octonion sub-algebra

The starting point is the standard electromagnetism-coupled Dirac equation,

$$\left[\gamma^\mu \left(\hat{p}_\mu - eA_\mu\right) - m\right] \Psi = 0, \tag{4.1}$$

with $c = \hbar = G = 1$, written out explicitly (no implicit summation) in terms of the spacetime derivatives $\partial_\mu \equiv \partial/\partial x^\mu$ and the four components Ψ_ρ of the wave function:

$$\sum_{\mu=0}^{3}\sum_{\nu=0}^{3}\Big(i\,[\gamma^\mu]_{\rho\sigma}\,\partial_\mu - [\gamma^\mu]_{\rho\sigma}\,eA^\mu\Big)\,\Psi_\sigma - \delta_{\rho\sigma}\,m\,\Psi_\sigma = 0. \tag{4.2}$$

The eight bases $i\gamma^\mu$ and γ^μ ($\mu = 0, 1, 2, 3$), together with the identity, satisfy three algebraic rules — linear independence, anticommutation of the γ^μ among themselves, and the cross-relations between the γ^μ and the $i\gamma^\mu$. The construction now identifies these eight bases with eight of the sixteen complex-octonion units. Concretely, the paper defines a sixteen-component "operator" object $\nabla^{\mathrm{EM}}_{\mathrm{con16}}$ carrying the ∂_μ on one set of sub-algebra bases and the eA^μ on a parallel set, together with a sixteen-component wave function $\Psi^{\mathrm{EM}}_{\mathrm{con16}}$. The Dirac equation with field then reads

$$\big(\nabla^{\mathrm{EM}}_{\mathrm{con16}} - m\big)\,\Psi^{\mathrm{EM}}_{\mathrm{con16}} = 0. \tag{4.3}$$

At $A_\mu = 0$ this reduces to the field-free split-octonion Dirac equation of [Koepl2006]; the proof that the field-coupled form is equivalent to the standard physics description proceeds by the gauge-transformation ansatz

$$\Psi^{\mathrm{EM}}_{\mathrm{con16}} \to \Psi^{\mathrm{EM}}_{\mathrm{con16}}\,e^{\,i_0\,\chi}, \qquad \partial_\mu\chi := eA_\mu, \tag{4.4}$$

with i_0 the distinguished non-real square root of $+1$ that generates the complex octonions from the octonions. The gauge transformation acts on the kinetic-operator bases through the algebra of i_0 and produces exactly the field couplings on the matching set of bases. The construction is thus the standard electromagnetism-coupled Dirac equation, written in complex-octonion arithmetic.

5 The same field, on the octonion sub-algebra

By analogy, the same field A_μ can be embedded into the **octonion** sub-algebra under exactly the same algebraic rules, defining

$$\big(\nabla^{\mathrm{Gr}}_{\mathrm{con16}} - m\big)\,\Psi^{\mathrm{Gr}}_{\mathrm{con16}} = 0. \tag{5.1}$$

The field-free limit of this relation is the **octonion Dirac equation** of [Koepl2007a] — the construction with Euclidean mass-shell, time-reversal symmetry of the propagator, and no distinguished forward direction. Adding A_μ via the same gauge-transformation ansatz $\Psi \to \Psi e^{i_0\chi}$ gives a field-coupled octonion Dirac equation in which A_μ now plays the role of a generator of the gravity-candidate force.

A structural caveat the paper itself notes. This construction implies that any spin-$\frac{1}{2}$ particle with non-zero rest mass m also carries an electric charge $e \neq 0$ — both forces are coupled through the same A_μ. This holds for charged Standard Model fermions (electron, muon, tau, the quarks and their antiparticles) but **would not be compatible with massive neutrinos**: a neutrino that carries rest mass but no electric charge cannot be sourced by the construction, since the gravitational coupling here is wired through the same A_μ that the electric charge couples to. We acknowledge that **neutrino oscillations are observed**; whether they require a non-zero rest mass or admit a different, mass-free mechanism is a separate question, and this digest takes no position on it. The construction here is straightforwardly compatible with massless neutrinos that oscillate by some other means; massive neutrinos would call for a separate algebraic backdrop in which gravity is wired together with the weak interaction rather than with electromagnetism.

6 The mixing angle α

With the two embeddings on the table, the unified construction comes from interpolating them on the ambient complex octonion. Following the same three-step recipe used in Paper 1 of the series [Koepl2006], the operator is split into two pieces — **a piece ∇_{Q1} that looks the same in both the split-octonion (Minkowski) and octonion (Euclidean) geometries, and a piece ∇_{Q2} that carries the signature distinction between the two**. Concretely (paper eqs. 17–18), across the sixteen complex-octonion basis slots $(1,\, i_1, \ldots, i_7,\, i_0,\, e_1, \ldots, e_7)$:

$$\nabla_{Q1} := (0,\, \partial_0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, eA_0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0), \tag{6.1}$$

$$\nabla_{Q2} := (0,\, 0,\, 0,\, 0,\, 0,\, \partial_3,\, \partial_2,\, \partial_1,\, 0,\, 0,\, 0,\, 0,\, 0,\, eA_3,\, eA_2,\, eA_1). \tag{6.2}$$

∇_{Q1} holds the time-direction derivative ∂_0 and the time-component gauge field eA_0 on basis elements that read the same way in either signature. ∇_{Q2} holds the spatial-direction derivatives $\partial_{1,2,3}$ and the spatial gauge fields $eA_{1,2,3}$, on the basis elements where the split-octonion-vs-octonion sign difference lives. The phase factor is then attached only to the signature-carrying piece:

$$\nabla^{\text{Gr,EM}}_{\text{con16}} := \nabla_{Q1} + e^{i_0 \alpha}\, \nabla_{Q2}, \tag{6.3}$$

with $\alpha \in \mathbb{R}$ a real **mixing angle**. The factor $e^{i_0\alpha}$ rotates the signature-carrying spatial sector through the multiplication table of the complex octonions, mediating **a continuous transition between the Euclidean and Minkowski metrics** without introducing any new field, mode, or coordinate substitution. An identical construction with parallel definitions of Ψ_{Q1} and Ψ_{Q2} (paper eqs. 20–22) gives the wave function $\Psi^{\text{Gr,EM}}_{\text{con16}}$. The unified equation is

$$\left(\nabla^{\text{Gr,EM}}_{\text{con16}} - m\right) \Psi^{\text{Gr,EM}}_{\text{con16}} = 0. \tag{6.4}$$

Two limits are immediate:

- $\alpha = \pi/2$: the construction reduces to **pure electromagnetism** — the Dirac equation with EM field on the split-octonion sub-algebra of the previous section.
- $\alpha = 0$: the construction reduces to **pure quantum-gravity-candidate** — the field-coupled octonion Dirac equation.

For intermediate α the two are mixed.

The decisive observation is that the complex-octonion modulus $|\Psi^{\text{Gr,EM}}|$ depends on α — a change in α generally changes the modulus. Demanding that physical law be the same in equivalent frames (the invariant-modulus relativity principle of [Koepl2007c]) therefore forces α to be constant under spacetime coordinate transformations. **The mixing angle α is a constant of the particle.** A particle in this picture is fully characterized by the triple

$$(m,\, e,\, \alpha), \tag{6.5}$$

three real parameters: rest mass, electric charge, and gravitational mixing angle. The paper does not derive α from first principles; it is left as a free parameter of the model, to be determined per particle.

7 The line element

To check what observers see, the same invariant-modulus construction is applied to a spacetime line element. Splitting $\mathrm{d}s_{\text{con16}}$ into a time piece $\mathrm{d}s_{Q1}$ (carrying $\mathrm{d}x_0$ on one sub-algebra basis) and a space piece $\mathrm{d}s_{Q2}$ (carrying $\mathrm{d}x_1, \mathrm{d}x_2, \mathrm{d}x_3$ on the parallel set), the unified line element is

$$\mathrm{d}s_{\text{con16}} := \mathrm{d}s_{Q1} + e^{i_0\alpha}\,\mathrm{d}s_{Q2}. \tag{7.1}$$

Computing the complex-octonion modulus directly — using the Carmody–Musès formula for the sedenion modulus [Carmody1988, Muses1980, Carmody1997] — gives the clean identity

$$|\mathrm{d}s_{\text{con16}}|^4 = \mathrm{d}x_0^4 + |\mathrm{d}\mathbf{x}|^4 + 2\,\mathrm{d}x_0^2\,|\mathrm{d}\mathbf{x}|^2\,\cos(2\alpha). \tag{7.2}$$

Two limits:

- $\alpha = \pi/2$: $\cos(2\alpha) = -1$, so $|\mathrm{d}s|^4 = (\mathrm{d}x_0^2 - |\mathrm{d}\mathbf{x}|^2)^2$ and $|\mathrm{d}s|^2 = \mathrm{d}x_0^2 - |\mathrm{d}\mathbf{x}|^2$ — the **Minkowski line element**.
- $\alpha = 0$: $\cos(2\alpha) = +1$, so $|\mathrm{d}s|^4 = (\mathrm{d}x_0^2 + |\mathrm{d}\mathbf{x}|^2)^2$ and $|\mathrm{d}s|^2 = \mathrm{d}x_0^2 + |\mathrm{d}\mathbf{x}|^2$ — the **Euclidean 4-space line element** of the projection ansatz.

For intermediate α, the line element does *not* reduce to a second-order tensor expression $g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$ at all — and a fourth-order tensor expression doesn't help either, because conic sedenions are non-associative in the relevant slots. **At intermediate α the standard tensor-algebra description of physics breaks down**, and one has to compute directly with the complex-octonion modulus.

8 The Standard Model approximation

For Standard Model particles the relative strength of the gravitational force is dozens of orders of magnitude weaker than the electromagnetic. Setting $\beta := \pi/2 - \alpha$ and Taylor-expanding around $\alpha = \pi/2$, the SM regime sits at

$$\beta^2 \sim \frac{F_{\text{Gr}}}{F_{\text{EM}}} \sim \frac{m^2}{e^2} \quad \text{(natural units)}. \tag{8.1}$$

For two electrons this is $\beta_e^2 \sim 10^{-42}$.

In this regime, the line element (7.2) expands to

$$|\mathrm{d}s|^2 \approx \mathrm{d}x_0^2 - |\mathrm{d}\mathbf{x}|^2 + \mathcal{O}\big(\beta^2\gamma^2\big)\,|\mathrm{d}\mathbf{x}|^2, \tag{8.2}$$

with $\gamma = (1 - |\mathbf{v}|^2)^{-1/2}$ the ordinary Lorentz factor. Even at the highest accelerator energies ever reached ($\gamma \sim 10^3$ at the Tevatron with protons, $\gamma \sim 2 \times 10^5$ at LEP with electrons), the correction $\beta^2\gamma^2$ is at most of order 10^{-29}. For everyday physics it is invisible. The construction predicts that ordinary Special Relativity continues to hold to extraordinarily high precision.

Genuine complex-octonion arithmetic — where the second-order tensor form fails — is only required at energies for which $\beta\gamma$ becomes order unity. For Standard Model particles this is the **Big Bang regime**, far above any experimental reach. The construction is offered as a tool for exploring **bound quantum states** in this regime: hypothetical particle types with $\alpha < \pi/4$ (predominantly gravitational coupling) that could form deeply bound configurations whose constituents are not directly observable.

9 From mixing angle to gravity phase

Sixteen years on, the mixing angle α of this paper is *one possible* algebraic origin of what [Koepl2023] calls the **gravity phase** — a phase inside the multiplication table of an ambient algebra that mediates between Lorentzian and Euclidean physics. The 2023 paper stands on its own: it retains a phase parameter as the central physical handle but simplifies the ambient from the 16-dimensional complex octonions down to the 4-dimensional bicomplex numbers $\mathbb{C} \oplus \mathbb{C}$, and provides phenomenological support for the present construction *together with* several other possible algebraic origins it cites in turn. With the phase kept as a free parameter, that paper computes Born-approximation Rutherford scattering of spin-$\frac{1}{2}$ particles on a fixed target and extracts phenomenological signatures: enhanced elastic backscattering at high energies, non-zero scattering cross section in the high-energy limit, and small-but-finite momentum transfer in known processes (neutrinos through matter, scattering in intergalactic gas).

The thread is therefore: **2007 introduces α as a third particle property in an algebraic-modeling framework; 2023 turns α into a quantum-gravitational degree of freedom with computable scattering predictions in a much smaller algebra.** The present paper offers one of the possible algebraic origins for that phase.

10 What the construction claims (and doesn't)

The paper is candid about scope.

What it claims.

- A coherent, closed algebraic framework that integrates the electromagnetism-coupled Dirac equation and a quantum-gravity candidate force on the same hypernumber ambient.
- A line element with two physical limits (Minkowski / Euclidean) and a controlled mixing in between.
- A new particle parameter α that joins m and e, with a clear connection to the gravitational-vs-electromagnetic force ratio.
- An estimate that all currently accessible experimental energies see only a tiny correction to ordinary Special Relativity, well below sensitivity.

What it does not claim.

- It does not predict α from first principles. The mixing angle is a free parameter per particle.
- It does not derive Standard Model spectra; the Standard Model is an *input* (through the values of m, e, and the ratio m^2/e^2).
- It is not compatible with massive neutrinos. The construction implicitly requires every massive spin-$\frac{1}{2}$ particle to carry electric charge, since gravity is wired through the same A_μ as electromagnetism. The construction takes no position on whether observed neutrino oscillations require a non-zero rest mass; if they do not, the construction is straightforwardly compatible with massless oscillating neutrinos.

- It does not provide a direct experimental signature at currently accessible energies; signatures only emerge at extreme energies (Big Bang regime) or — as the 2023 follow-on shows — through precision scattering observables that the present paper does not itself compute.

The paper's outlook section also notes prospects for extending the invariant-modulus construction to **higher hypernumber levels** beyond the complex octonions, with an eye toward modeling the weak interaction (via w-arithmetic anti-commutative cycles reminiscent of $SU(2)$); this remains speculative and is not developed in the paper.

11 How to read the paper

Six pages, formula-driven. Recommended order:

- **§1 Introduction.** Sets the four-paper context.
- **§2 Electromagnetism.** Adds A_μ to the split-octonion Dirac equation; equivalence with standard physics is by direct gauge transformation in complex-octonion arithmetic.
- **§3 Gravity.** Adds the same A_μ to the circular Dirac equation.
- **§4 Mixing gravity and electromagnetism.** Introduces the mixing angle α; derives the line element and its SM-limit approximation. This is the section worth reading carefully — it contains the physical content of the paper.
- **§5 Conclusion and outlook.** Mentions related work (octonionic electrodynamics [Gogb2006]; Hawking on Euclidean quantum gravity) and speculates about higher hypernumber levels.

Footnote 2 of the paper notes a sign-correction to definitions (3) and (4) of [Koepl2006]; this is also documented in the 2007 corrigendum [KoeplCorr2007].

12 About this digest

For preprints and personal versions of the author's papers, see the author's web page at [KoeplWWW]. The author's ResearchGate profile is [ResearchGateJK].

13 How to cite this digest

J. Köplinger, "*Gravity and electromagnetism on conic sedenions* — a complex octonion phase α as particle property", research digest (2026). Available via ResearchGate.

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