Signature of gravity in conic sedenions — a guided tour of the complex-octonion gravity phase

A research digest of Köplinger (2007)

Abstract

This is a guided tour of the paper **J. Köplinger, "Signature of gravity in conic sedenions", *Appl. Math. Comput.* 188 (2007) 942–947** (DOI: 10.1016/j.amc.2006.10.049). The paper takes the split-octonion Dirac equation $\nabla \cdot \Psi = 0$ of Köplinger (2006) and embeds it in the sixteen-dimensional complex octonions, where a one-parameter rotation by an angle α in the $(1, i_0)$ plane interpolates between a Minkowski-signature (split-octonion) and a Euclidean-signature (octonion) Dirac equation. Eigenvectors and the Green's function of the new, Euclidean form are worked out, and the qualitative hallmarks one would expect from a gravity-like primitive — time-reversal symmetry, Euclidean mass-shell, no distinguished forward direction — are shown. The rotation is a genuine internal-algebra wiring, not a single-element Wick substitution. The digest places the construction in the context of the author's 2023 *gravity phase* paper on bicomplex numbers and in the wider modern literature on Euclidean-to-Lorentzian mechanisms in gravity.

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1 At a glance

This paper takes the split-octonion Dirac equation of Köplinger, *Appl. Math. Comput.* **182** (2006) 443–446 (DOI: 10.1016/j.amc.2006.04.005) [Koepl2006], embeds it in the **complex octonions** — a sixteen-dimensional non-associative algebra with a distinguished non-real square root of $+1$, written here i_0 — and rotates it by an angle α in the $(1, i_0)$ plane. At $\alpha = \pi/2$ one gets back the original split-octonion (Minkowski-signature) Dirac equation. At $\alpha = 0$ one gets a new **circular Dirac equation** on the ordinary octonions with a **Euclidean** signature. The paper writes down the eigenvectors and Green's function of the circular form, and points to the qualitative hallmarks of a gravity-like primitive — the "signature of gravity" of the title.

The rotation in $(1, i_0)$ is what this digest calls the **gravity phase**. It is a one-parameter hypernumber rotation inside the multiplication table of the ambient algebra — the same construction that, sixteen years later, reappears in simpler form on the bicomplex numbers [Koepl2023], and is the reason this short paper is worth revisiting today.

2 Who this digest is for

Readers who have already met the split-octonion Dirac equation in [Koepl2006] and its companion digest. That digest explains the algebra, the $\nabla \cdot \Psi = 0$ identity, and the naming conventions used here; this one builds on it directly, without repeating that material.

3 Note on terminology

As in [Koepl2006], the original uses the *Musean hypernumber* vocabulary in exchange at the time of writing:

Name used in the paper	Conventional modern name
conic sedenion	complex octonion
hyperbolic octonion	split-octonion
circular octonion	octonion

The two vocabularies are synonymous — the mathematics is the same algebra under either name. This digest uses the conventional names throughout.

4 The setup

The split-octonion Dirac equation of [Koepl2006] is the relation

$$\nabla_8 \cdot \Psi_8 \;=\; 0, \tag{4.1}$$

in which ∇_8 and Ψ_8 are eight-component split-octonions carrying the mass, derivatives, and wave-function components of the standard Dirac equation on Minkowski signature $(+, -, -, -)$.

The complex octonions form a sixteen-dimensional algebra spanned by the eight octonion units and their images under multiplication by i_0, a non-real element satisfying $i_0^2 = +1$ that commutes with everything. One can embed any eight-dimensional split-octonion object on the "real" half of the complex octonions; the "i_0" half is then available as an adjacent eight-dimensional sector.

With this in hand, the paper writes a single sixteen-component relation

$$\nabla_{16} \cdot \Psi_{16} \;=\; 0, \tag{4.2}$$

and introduces a rotation in the $(1, i_0)$ plane by the phase α, acting on the algebra multiplication table. At $\alpha = \pi/2$ this relation *is* the split-octonion Dirac equation; at $\alpha = 0$ it becomes a new relation on the ordinary octonions, dubbed the **circular Dirac equation**.

The rotation by α is not a Wick rotation. Wick rotation substitutes a single element ($t \to it$, or equivalently the "*ict*" device): it replaces one coordinate or one basis vector. The gravity phase α, in contrast, *rotates the spacetime basis through the internal wiring of the algebra itself* — inside the complex-octonion multiplication table — and emerges with a different signature on the other side. That distinction is the point of the construction and is what the rest of the paper probes.

5 The circular Dirac equation

Written as a 4×4 matrix relation on four complex-valued wave-function components (for analytical convenience only — the authentic formulation is the hypernumber identity above), the circular Dirac equation differs from the standard Dirac equation only by selected sign changes on its diagonal and off-diagonal entries. The paper verifies the circular form component-by-component by unpacking the complex-octonion product.

What one should take away:

- **Same shape, different signs.** The circular Dirac equation is a close relative of the ordinary Dirac equation — not a rewriting of it, but a cousin one rotation away.
- **Pure Euclidean signature.** The sign changes are exactly those needed to move from Minkowski $(+,-,-,-)$ to Euclidean $(+,+,+,+)$ behavior in the anticommutator algebra of the Dirac gamma matrices.

6 Eigenvectors and time-reversal symmetry

The paper solves for the plane-wave eigenvectors of the circular Dirac equation. It finds two pairs, $\Psi_1^{\pm}$ and $\Psi_2^{\pm}$, that carry the expected $\exp(i(\mathbf{p}\cdot\mathbf{x} \pm E\,t))$ factors. Unlike the ordinary Dirac equation, the eigenvectors for $+$ and $-$ energies are *not* distinguished by the structure of their four-component spinor — only by the sign in the plane-wave factor, together with a permutation of indices. That is, the circular Dirac equation is symmetric under time reversal in a way the ordinary Dirac equation is not.

This is exactly the symmetry one expects from the Euclidean signature of the previous section: in Euclidean four-space there is no distinguished time axis, so past and future play the same role.

7 The Green's function and the Euclidean pole

The paper sets up the Green's function of the circular Dirac equation by Fourier transform in the usual way. The resulting propagator has a pole at

$$m^2 \;=\; E^2 + |\mathbf{p}|^2, \tag{7.1}$$

an all-plus Euclidean sphere, in place of the familiar Minkowskian mass-shell $m^2 = E^2 - |\mathbf{p}|^2$. The anticommutator of the associated β-matrices reduces to $\delta_{\mu\nu}$ rather than the Minkowski $\eta_{\mu\nu}$, confirming the Euclidean metric from a second direction.

Transformed back to position space, the Green's function integrates to an expression that is symmetric in all four coordinates: no retarded / advanced split, no light cone, no distinguished forward direction of time. The "signature of gravity" of the title refers to this collection of features: **Euclidean metric, time-reversal symmetry of the propagator, no light cone** — qualitative hallmarks one would expect from a gravity-like, rather than a matter-like, primitive.

8 The gravity phase, then and now

At the time of writing (2006–2007), what this paper calls an α-rotation between circular and split-octonion Dirac forms was a small, specific observation: the continuous family of relations $\nabla\cdot\Psi = 0$

on the complex octonions, parametrized by $\alpha \in [0, \pi/2]$, interpolates between a Euclidean regime and a Minkowski regime without adding a field or a mode or a degree of freedom. The rotation lives entirely inside the algebra.

Sixteen years later the same idea reappears in a much simpler setting — a 4×4 complex-matrix Dirac equation over the bicomplex numbers $\mathbb{C} \oplus \mathbb{C}$ — with enough structure to carry through a spin-$\frac{1}{2}$ Coulomb scattering calculation in the Born approximation and compare it to General Relativity predictions [Koepl2023]. That paper gives the phase α its standing name — the **gravity phase** — and frames it as a phenomenological handle for a wide class of "complexified spacetime" models in which Euclidean and Minkowskian regions coexist.

What the present paper contributes to that line is the **complex-octonion simplicity** of the α-phase construction. The phase does not act on one coordinate or one basis vector; it acts on the whole spacetime basis through the internal wiring of an ambient non-associative algebra whose multiplication table happens to contain both a split-octonion and an octonion sub-algebra. That the same algebra accommodates both signatures, joined by a single continuous angle, is what the construction is saying.

9 Prior art, parallel art, and how the present approach sits

There are many routes to introducing a Euclidean-to-Lorentzian transition in physics — from Wick rotation as a calculational trick, to signature-changing spacetimes, to gauge-theoretic constructions, to disformal metric relations driven by a scalar or vector field. For a recent survey of the dynamical end of that spectrum — models that take a fundamentally Euclidean theory and recover Lorentzian physics through a field-theoretic mechanism — see Koivisto, Zheng, and Zlosnik [KZZ2025] and its bibliography. Among the works cited there, the closest in spirit to the present phase construction are Girelli, Liberati, and Sindoni [GLS2009]; Mukohyama and Uzan [MU2013]; Svidzinsky [Svidz2017]; Nash [Nash2023, Nash2025]; and Feng, Mukohyama, and Carloni [FMC2025].

These works introduce a Euclidean-to-Lorentzian phase through various devices: a scalar field gradient, a disformal metric, a "khronon" field, a gauge construction. Each is worth reading on its own terms, and each answers a different question.

The contribution of the present paper — and of the line of work that extends from it — is narrower, and different in kind: the phase is already there in the multiplication table of an ambient number system. No field, no device, no extra structure is introduced; a single internal angle α in an ambient algebra mediates between two signatures. That is a modest offering, and a concrete one. The aim is not to argue that this construction is preferable to any of the others, but to add one more route to the map, in case a future model finds a use for it.

10 How to read the paper

The paper is short (six pages). The recommended reading order is:

1. Skim §1 for the motivation and the $(1, i_0)$ rotation idea.
2. Read §2 closely — this is where the circular Dirac equation is written down and where most of the algebraic content lives.
3. Read §3 and §4 in that order; §3 gives the physical content (eigenvectors, time-reversal), §4 gives the mass-shell and propagator structure that carries the Euclidean signature.

4. §5 is a one-page conclusion with open questions; these map naturally onto [Koepl2023].

A reader who wants the full mechanical derivation of the complex-octonion embedding — and, in particular, the sign-corrected form of the 2006 identity on which this paper builds — should read this paper alongside [Koepl2006] and its corrigendum [KoeplCorr2007].

11 About this digest

For preprints and personal versions of the author's papers, see the author's web page at [KoeplWWW]. The author's ResearchGate profile is [ResearchGateJK].

12 How to cite this digest

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