Emergent Time from Non-Associative Quantum Theory

Unobservability, quantum gravity, and hints towards unifying space-time and isospin symmetries with octonionic algebras

Jens Köplinger, Vladimir Dzhunushaliev and Merab Gogberashvili
14 November 2008

> *"And an astronomer said, Master, what of Time? And he answered: You would measure time the measureless and the immeasurable."*
>
> Kahlil Gibran, "The Prophet"

A good friend once had a digital clock whose battery went dead, but it always gave the correct time. How? one may ask. "Well," he said, "I wrote on it, in digital font: N-O-W ! — the only time there is!" In this view, you can't make up some container whose true nature would be measurable with a chronometer. Instead, *time* is part of a fundamental intellectual structure, which can only be experienced subjectively.

Physics, in contrast, describes time more like frames on a film strip, on which events occur in sequence. The time line resembles a container, if abstract, on which one could conceptually travel back and forth, to trace evolution of events from an outsider's, objective point of view.

Simultaneity is not a global concept

Proper time from Einstein's relativity is a locally measurable, directed axis that puts events in space-time into causal relation. The film strip analogy, however, doesn't hold much further: Simultaneity, and mere sequence of events outside the light cone, are not global concepts anymore; different observers generally cannot agree on simultaneity in principle. This may be irritating for the objectively-minded amongst us, who are being denied "the full view" on reality. But the success of this model is rather undeniable: No experiment to-date has shown any different.

"N-O-W ! — the only time there is!"

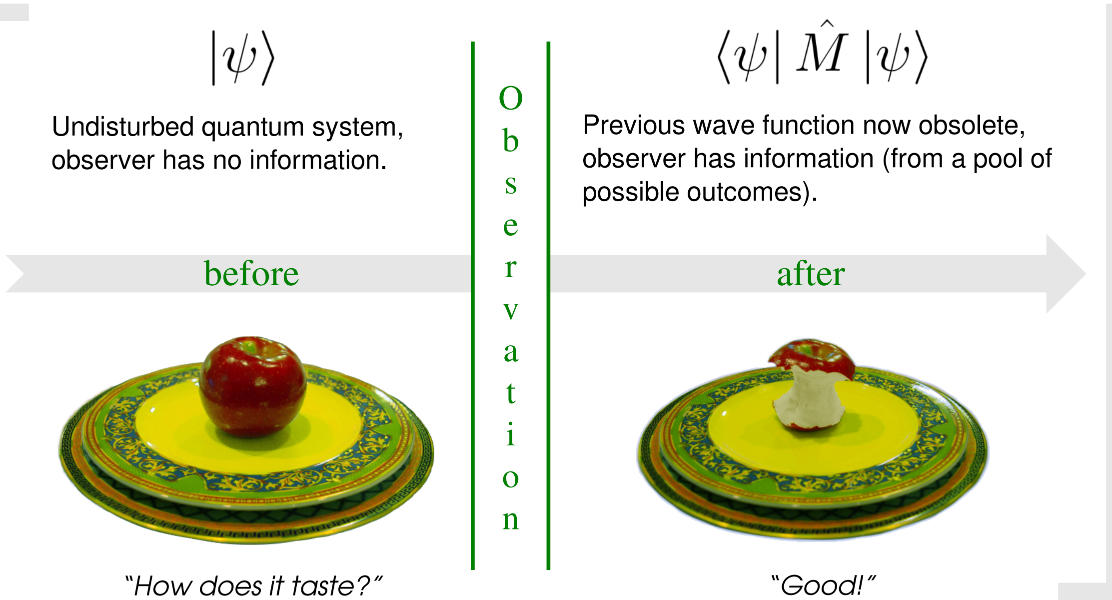
This space-time is then warped by gravity some more, and the Standard Model describes the weak and strong forces through isospin symmetries intrinsic to any point. Physicists predict current experimental outcomes within their precision limits — but acknowledge conceptual incompatibility of the models required to make these predictions.

Fundamental research, or irrational optimism?

Gifted with seemingly irrational optimism towards finding a simpler, more coherent description of the forces of nature, physicists feverishly work on many approaches, hoping one may eventually lead the way.

In this essay, we highlight a recent effort, non-associative quantum mechanics, and discuss *time* as an emergent parameter from a wider, general background. We point to an implied conceptual time asymmetry from traditional observation, and distinguish *unobservables* from hidden variables. Octonionic algebras can model such unobservables, together with space, time, Heisenberg uncertainty, and quantum gravity.

A new unified treatment of isospin and space-time symmetries will be proposed, where observables can no longer be distinguished from the parameters which describe them. In this proposal, time would emerge as a parameter only when asking a quantum system for information, whereas a speculated master symmetry would not be built on a space-time (or higher dimensional) manifold anymore.



A fundamental time asymmetry from observation that requires irreversible digestion of information.

Time asymmetry from observation

Asking for the result of an observation assumes that information is not available to the observer beforehand, but will be available afterwards. It implicitly introduces a time asymmetry, between the *before* and the *after*.

If observation leaves the observed unchanged, one could conceivably reverse the direction of time, play events backwards, and find observer and observed somewhat dual to each other. However, if observation is irreversible as in quantum mechanics, this thought experiment is in vain: Extraction of information from a quantum system is fundamentally asymmetric in time, the information obtained does not allow to reconstruct the initial state of the system. There are no "hidden variables" that would allow an observer to gain full knowledge about a quantum system, to make definitive predictions on measurement outcomes under identical circumstances.

Anthropocentric bias?

In order to lay out a basis for describing physical law on non-associative algebras, the following key characteristics of observation in quantum mechanics will now be put into question: Time-asymmetry, indeterminism and uncertainty.

We will make the hypothesis that these characteristics are the result of an unnatural, human-centric approach to quantum behavior: Asking for the result of an observation is simply the wrong question.

One might speculate that irreversible observation happens exactly when the desired information is not in the nature of the observed. As an example, we might know all the molecules in an apple, have full knowledge about the human sense of taste; we might have modeled what we believe to be *the* perfect apple — yet we cannot predict with certainty what your neighbor will say when asked the simple question: "How does it taste?" If we are interested in the inner structure of the apple, then we have asked the wrong question …

The problem at hand

Classical quantum mechanics models observation with wave functions that predict measurement outcomes through well-defined probability ranges, but are intrinsically indeterministic. Toward the interface to the non-quantum world, events become causally related within the light cone, and therefore increasingly deterministic. This change in physical behavior can be parametrized through Heisenberg uncertainty, i.e., to probability ranges that describe how well certain properties of a quantum system may be measured simultaneously. From a human's point of view, the physical world is "pretty much" deterministic, with a well-understood variance.

To support the hypothesis that this traditional approach to observation in quantum mechanics is unnatural, the concept of *unobservables* [1] will now be used: Modeled by non-associative parts of quantum mechanical operators, their measurement outcomes cannot be predicted in principle.

Well-defined, but unobservable in principle

On first sight, the concept of unobservability may seem rather philosophical: What is gained by unobservable parameters, as compared to simply assuming that measurement outcomes are indeterministic?

Our hypothesis that time-asymmetry, indeterminism, and uncertainty are a product of the traditional approach to quantum mechanics, based on irreversible observation, must ultimately prove its validity by describing known physical law using fewer assumptions — a daunting task, when looking at the successes of General Relativity and the Standard Model.

Findings to-date nevertheless indicate that searching for physics on non-associative algebra is indeed a serious endeavor: Three space-like and one time-like dimension emerge from description of classical electromagnetism on non-associative algebra, it relates the light cone to Heisenberg uncertainty, and quantum gravity can be unified with electromagnetism.

Granted, “first quantization” may not be the hottest discussion on the planet today — but we are excited about the outlook on how operator quantum mechanics could conceivably unify all known forces, on a background that does not distinguish between observables and the parameters used to describe them.

Non-associativity and unobservability

Familiar matrix multiplication is generally non-commutative ($ab \neq ba$) but associative: The product of any three elements *a*, *b*, and *c* evaluates to the same result by pairwise multiplication: $(ab)c = a(bc)$. This is not the case anymore for non-associative algebra, where generally

$$(ab)\,c \neq a\,(bc)\,. \tag{1}$$

This cannot be modeled anymore by matrix algebra over real or complex valued coefficients, and has far reaching consequences for non-associative parts of quantum mechanical operators [1].

If M is eigenvalue of an operator,

$$M\left|\psi\right\rangle = \hat{M}\left|\psi\right\rangle, \tag{2}$$

the expectation value over some space V would be:

$$\langle M\rangle = \int \psi^* \hat{M}\psi\, dV. \tag{3}$$

Allowing operators and wave functions on non-associative algebra, however, this expression becomes ambiguous:

$$\left(\psi^* \hat{M}\right)\psi \neq \psi^*\left(\hat{M}\psi\right). \tag{4}$$

Not being able to determine the expectation value of an operator makes it inherently unobservable: The measurement outcome cannot be predicted.

Specifically, looking at the time evolution of a quantum mechanical system,

$$\hat{M} := \exp\left(i\hat{H}t\right), \tag{5}$$

a non-associative part in the Hamiltonian may be interpreted as not preserving probability over time. We have lost the ability to forecast behavior of the system.

Octonions

From the seemingly infinite number of non-associative algebras that may exist, it is prudent to find one which is as similar as possible compared to what's in use today: We are already able to explain experimental outcomes very well, using associative algebra.

Arguably, this algebra should be some structure that models dimensions on scalable axes, permits basic arithmetic including division, and has a concept of magnitude or distance that satisfies the triangle inequality:

a normed division algebra with non-associative multiplication, that is a vector space over the reals. As it so happens, there is only one such algebra that does the trick: The octonions.

Octonions are an eight-dimensional number system, with one real and seven imaginary axes. Written to a basis

$$b_{\mathbb{O}} := \{1, i_1, \ldots, i_7\}\,, \tag{6}$$

the imaginary basis elements are anti-commutative,

$$i_n i_m = -i_m i_n \qquad (n \neq m) \tag{7}$$

and are anti-associative exactly when any product of three different elements does not form a 3-cycle:

$$\begin{aligned} (i_n i_m)\, i_l \;&=\; -i_n\,(i_m i_l) \\ &\Updownarrow \\ i_n i_m \;&\neq\; \pm i_l \qquad (n \neq m \neq l) \end{aligned} \tag{8}$$

The associative 3-cycles can be chosen, e.g. as:

$$i_m i_n \;=\; \epsilon_{mnl} i_l - \delta_{mn} \tag{9}$$

with the totally anti-symmetric Levi-Civita symbol

$$\epsilon_{mnl} \;=\; +1 \tag{10}$$

where

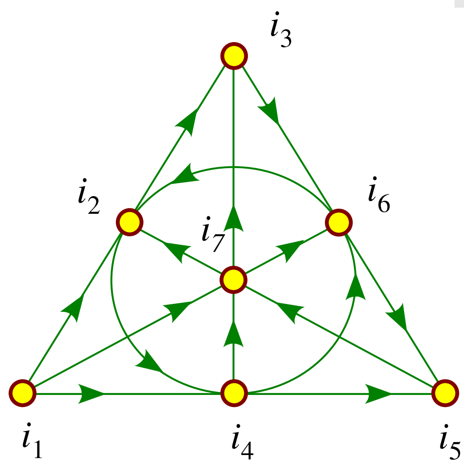
$$mnl \;\in\; \{123, 145, 176, 246, 257, 347, 365\}\,. \tag{11}$$

A typical visual representation of the product of octonion basis elements is the *Fano plane* (pictured). There, any three basis elements on a straight line (and the center circle) form an associative 3-cycle. The arrows indicate the sign of the anti-commutative product.

A notable sub-algebra, next to the reals and complexes, are the *quaternions*, which can be obtained by choosing one real and three associative imaginary axes, e.g.

$$b_{\mathbb{Q}} := \{1, i_1, i_2, i_3\}\,. \tag{12}$$

Octonion multiplication in the Fano Plane.



Select octonion properties

Of great use for the physics models referenced in this essay is the multiplicative norm of an octonion. Writing the real number coefficients as $\{x_0, x_1, \ldots, x_7\}$, the octonion norm $\|x\|$ is then defined as its Euclidean 2-form:

$$\|x\|^2_{\mathbb{O}} := x_0^2 + x_1^2 + \ldots + x_7^2 = \sum_{m=0}^{7} x_m^2. \tag{13}$$

This allows for an intuitive concept of “distance” in octonion space. Because the norm is also multiplicative,

$$\|x\|\,\|y\| = \|xy\| \tag{14}$$

the magnitudes associated with each of these numbers scale geometrically in a familiar way: Multiplication in octonion space is a vector rotation in eight dimensional space, where the length of a resulting vector is the product of the lengths from the initial factors.

The symmetries that govern octonion multiplication, and certain higher-dimensional octonionic constructs, are the *exceptional Lie groups* (G_2, F_4, E_6, E_7, and E_8). This and many other notable facts about octonions are contained in the comprehensive analysis [2] that earned this remarkable system its infamous title “the crazy old uncle nobody lets out of the attic”, due to its non-associativity.

From the crazy old uncle's lineage

Before – finally – writing about physics on non-associative algebra, there are two more number systems to be introduced: *Complex octonions*, and *split-octonions*.

Complex octonions are octonions supplied with complex valued coefficients (instead of real numbers). Split-octonions can then be defined as a closed sub-algebra of the complex octonions — while keeping in mind that several overlapping definitions exist, notably “Zorn's vector-matrix algebra” for split-octonions, and “conic sedenions” for complex octonions.

To keep this essay short and enjoyable, we will only give some highlights here, but not go into details regarding alternative definitions and programs. Instead, one may find the ongoing disambiguation effort in Wikipedia helpful (e.g. from “hypercomplex number”).

Writing complex valued coefficients to a basis

$$b_{\mathbb{C}} := \{1, i_0\}, \tag{15}$$

the complex octonions can be expressed with real number coefficients to basis elements:

$$b_{\mathbb{C}\otimes\mathbb{O}} := \{1, i_1, \ldots, i_7, i_0, \varepsilon_1, \ldots, \varepsilon_7\}. \tag{16}$$

Here, the new basis elements are defined as

$$\begin{aligned} \varepsilon_n &:= -i_0 i_n, \\ \varepsilon_n^2 &= +1, \end{aligned} \tag{17}$$

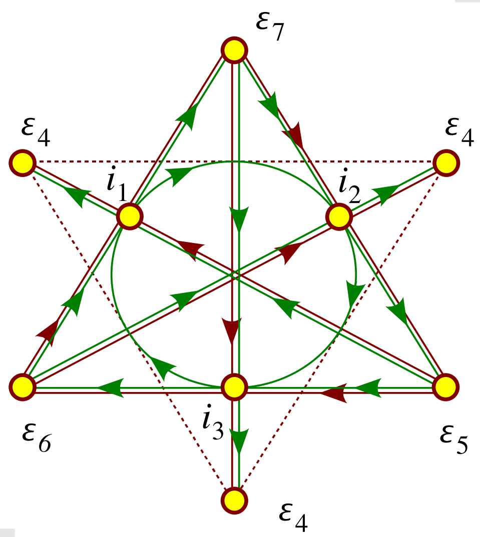
where i_0 is commutative and associative with all others:

$$\begin{aligned} i_0^2 &:= -1 \\ i_0 i_n &:= i_n i_0, \\ i_0 \varepsilon_n &:= \varepsilon_n i_0. \end{aligned} \tag{18}$$

This way, the basis for split-octonions can be written as:

$$b_{\text{split-}\mathbb{O}} := \{1, i_1, i_2, i_3, \varepsilon_4, \varepsilon_5, \varepsilon_6, \varepsilon_7\}. \tag{19}$$

Split-octonion multiplication as David's star; only $\{i_1, i_2, i_3\}$ is cyclic [3].



Just as the octonions, the split-octonions have a multiplicative 2-form; only this time it is not Euclidean, but hyperbolic, therefore hinting at Minkowskian geometry:

$$\|x\|^2_{\text{split-}\mathbb{O}} := x_0^2 + \ldots + x_3^2 - x_4^2 - \ldots - x_7^2. \tag{20}$$

Physics — finally! It's about *time* …

So far, we have introduced new algebras, new concepts, and a hypothesis that puts much of today's interpretation of quantum mechanics into question. We have argued that *unobservables* provide internal consistency, namely that the proposed tools to model new physics support the principles put into question.

Now, it needs to be proven that physical law can actually be formulated, using these new tools, to describe the reality we observe. The hyperbolic 2-form of split-octonions (20) looks suspiciously similar to Minkowskian space-time, therefore hinting at a Lorentz-invariant force: electromagnetism. And, alas, it is possible to model the Maxwell equations on an algebraic background described by split-octonions [4,5].

You may still believe that all of this is merely a new piece for the cabinet of curiosities of mathematical physics: interesting yes, but inconsequential. As it turns out, however, the additional degrees of freedom, and the new ways to manipulate formulations on non-associative algebra, have some intriguing surprises in store.

Invariant split-octonion length element

In its most general form, the invariant length element on split-octonions [3] can be written as:

$$s \;:=\; c\left(t+\varepsilon_4\hbar\omega\right)+\sum_{n=1}^{3}\left(\varepsilon_{(n+4)}x_n+i_n\hbar\lambda_n\right),$$

$$\|s\|^2 \;=\; \left[c^2t^2-|\vec{x}|^2\right]-\hbar^2\left[c^2\omega^2-\left|\vec{\lambda}\right|^2\right]. \qquad (21)$$

Here, time t and space x_n are paired with their canonical transforms: ω is of unit energy^{-1} and the λ_n are momentum^{-1}. Invariant properties from Minkowski space-time are apparent, and one time-like and three space-like dimensions appear in this algebra from the signature of its 2-form.

Similar to Special Relativity, only positive values of $\|s\|^2$ are actually observable; negative values would result in an imaginary square root, making its associated *proper time* τ non-real. Events would be placed outside the light cone, inaccessible by the observer.

Maximum velocity and Heisenberg uncertainty

Derived from these seemingly harmless definitions is an interesting surprise. When looking at the time evolution of the invariant split-octonion length element,

$$\frac{d\tau}{dt} \;:=\; \frac{d\sqrt{\|s\|^2}}{c\,dt} \qquad (22)$$

$$=\sqrt{\left[1-\hbar^2\left(\frac{d\omega}{dt}\right)^2\right]-\frac{|\vec{v}|^2}{c^2}\left[1-\hbar^2\left(\frac{d\lambda_n}{dx_n}\right)^2\right]},$$

using speed

$$\vec{v} \;:=\; \frac{d\vec{x}}{dt}, \qquad (23)$$

this expression evaluates to an observable magnitude exactly when the argument of the square root is non-negative:

$$|\vec{v}| \;\le\; c, \qquad \hbar \le \left|\frac{dt}{d\omega}\right|, \qquad \hbar \le \left|\frac{dx_n}{d\lambda_n}\right|. \qquad (24)$$

Each of these three conditions are quite familiar, but surprising in context: There is a speed limit c, variations in time and energy must be bigger than (or equal to) the Planck constant, and variations in space and momentum must also be bigger than $\hbar$. In other words, in split-octonion space the existence of a maximum velocity has the same geometrical meaning as Heisenberg uncertainty.

Split-octonion roundup

Findings so far appear well-aligned with our hypothesis that time asymmetry and uncertainty in quantum mechanics arise from an unnatural approach, i.e., by asking for the result of an observation as a function of time: Requiring the split-octonion length element to be observable at all times (22), the light cone and Heisenberg uncertainty emerge as boundary conditions. Causal relation of events and parameter ranges of indeterminism appear on equal geometrical footing, in a compact algebraic model.

As an added bonus, split-octonions also explain the one time-like and three space-like dimensions that we observe; but to be honest, it is acknowledged that this property was the reason for looking at this algebra *a priori*. It would not be fair to tell the hen that the egg was first …

On a more subtle note, it is pointed out that s is made up from coordinates alongside dynamic parameters that characterize that same system. Planck's constant $\hbar$ acts as a proportionality factor. This is consistent with our speculation that non-associative algebra could eventually be used to model a background that does not distinguish between observables and the very parameters used to describe them.

Similar, but simpler: Gravity

Modeling electromagnetism on split-octonions is certainly a good start when investigating observability: Human perception of time and nature is dominated by electromagnetic interaction, between atoms, molecules, and light. All other experienced knowledge of the forces of nature consists of degenerate forms of the weak, strong, and gravitational interaction. We are aware of directed time, but hardly of the symmetries governing nuclear forces and particle decay. Near regions of high energy density, gravity warps the geometry of space and time beyond recognition; it is hard to visualize a force where sources are in dynamic balance with the fields they generate.

Nevertheless, looking at the fundamentals of gravity and comparing it to electromagnetism, the two forces appear quite alike — indeed, one may even argue that gravity is similar, but *simpler*: Both forces have a far field of infinite range, which decreases in strength with the inverse square of distance to a source. Under time reversal, electrical charges reverse sign, whereas gravity simply remains invariant. Electrical charge is a particle property separate from its mass, whereas in gravity simply *all* mass and energy is a source of the force.

Or more pointed, gravity doesn't have the complexity of opposing versus attracting charges. In fact, it doesn't require an extraneous concept (charge) at all: Gravity is universally attractive for all forms of energy.

Four dimensional Euclidean quantum gravity

How could one then model gravity in a formulation that is similar, but simpler as compared to electromagnetism? A clue comes from non-associative algebra: Split-octonions model electromagnetism. Therefore, the "simpler" octonions appear of interest for gravity.

Octonions have a Euclidean norm (13), and hint at well-established four dimensional *Euclidean quantum gravity* [6]. Since octonions and split-octonions both are subalgebras of complex octonions, it should just be a technicality to propose a unified description of the forces later on.

There is only one problem: Known Euclidean quantum gravity is a field theory on geometries that only indirectly relate to observer space-time; it can e.g. be modeled on anti-de Sitter spaces, which contain additional dimensions not apparent to humans. The "similar but simpler" proposition, on the other hand, would imply that the very same observer space-time is governed by two geometries simultaneously: Minkowskian for electromagnetism, and Euclidean for gravity. An obvious dilemma.

Back to the roots: operator quantum mechanics

The way out (at least as far as this essay is concerned) comes from an inconspicuous direction: operator quantum mechanics. Seemingly stowed away with other antiquated science wisdom, "first quantization" allows to unify electromagnetism and gravity, each corresponding to a distinct case where the generally non-associative mathematical description is reducible to associative matrix algebra (this and the following sections per [7]). Following our human-centric quest for "observation", wanting to extract information over time, we are bound to notice only these two discrete solutions on associative algebra. The wider, non-associative parts of the formulation remain unobservable to us, in principle.

And just in case you believe that this all is a futile attempt to teach an old dog new tricks, that quantum field theory would have yielded this opportunity much more efficiently than operator quantum mechanics, we would like to point out that classical quantum field theory assumes existence of a reasonably well formed base manifold, which ultimately corresponds to observer space-time. Such a base manifold, however, does not exist anymore in these models on non-associative algebra. Instead, observer space and time become locally emergent parameters that are generated by the aggregate effect from all interactions with that observer.

Euclid versus Minkowski, naturally aligned

The program's name, "naturally aligned elementary equations" (*NatAliE* equations), is straightforward: It pairs fundamental relations on Minkowskian space-time (such as invariant length element and Lorentz transformation) with direct counterparts on four dimensional Euclidean space-time. These are then interpreted as two distinct observable cases within a wider, unobservable context.

For the large body, non-quantum limit, consistency is demonstrated by projecting effects from such approach onto a purely Minkowskian observer: It proves to reproduce General Relativity. In turn, this can be embedded into a wider formulation on non-associative algebra, namely complex octonions.

A brief excursion with Natalie

The *NatAliE* equations proposition begins with the invariant length element and mass-energy-momentum relation on Minkowski space-time, and pairs them with the "similar but simpler" relation on Euclidean geometry.

The invariant length elements are:

$$\begin{aligned}(c\,dT)^2\Big|_{\text{Minkowski}} &= (c\,dt)^2 - |d\vec{x}|^2 \\ &\Updownarrow \\ (c\,dT)^2\Big|_{\text{Euclid}} &= (c\,dt)^2 + |d\vec{x}|^2\,.\end{aligned} \tag{25}$$

A bit unusual on first view maybe, the energy-mass-momentum relation compares as:

$$\begin{aligned}\left(mc^2\right)^2\Big|_{\text{Minkowski}} &= E^2 - |\vec{p}c|^2 \\ &\Updownarrow \\ \left(mc^2\right)^2\Big|_{\text{Euclid}} &= E^2 + |\vec{p}c|^2\,.\end{aligned} \tag{26}$$

Classical Lorentz transformation is then paired with a counterpart on Euclidean metric, to preserve invariant properties between unaccelerated frames of reference.

All of this is applied to the $1/r$ potential of a point mass, projected from four dimensional Euclidean space to Minkowskian observer space-time, and generalized to tensor formulation modeling arbitrary energy distributions: It becomes the linearized field equations from General Relativity. A known “bootstrap” method models self-interaction of the gravitational field, and it is concluded that the *NatAliE* equations indeed reproduce Einstein gravity in the semi-classical limit [7].

Mixing gravity with electromagnetism

While the non-quantum limit satisfies experimental requirements, this does not allow us to uniquely deduce the quantum mechanical description from which it emerged. Now closing the loop with the hypothesis of this essay, that time asymmetry, uncertainty, and indeterminism are the result of an unnatural approach to quantum mechanics, we argue:

Electromagnetism and gravity only appear as two distinct forces, because mixing effects are governed by unobservables, eluding traditional observation and extraction of information over time.

Unification of these forces was not successful in the past, because description in first quantization requires non-associative algebra — which had not been tried.

Using complex octonions, the proposed unification model follows parsimony, by requiring as few new concepts, assumptions, or parameters as possible.

The resulting formulation contains parts that are inaccessible to observation, and observer space and time are reinterpreted as emergent parameters.

It is pointed out that algebraic simplicity remains a key motivation for the approach: The experiment alone does not allow to distinguish *unobservables* from the indeterministic, “no-hidden-variables” interpretation of quantum mechanics. Explaining physical law with fewer assumptions is desirable in itself, and has traditionally led to prediction of new effects not envisioned at first.

A closer shave with Occam's razor

All this looks as follows: Take a four-potential A_μ, a charge e with mass m, and space-time derivatives ∂_μ on wave function components ψ_ν. Then, describe the most simple unification model that satisfies the conditions outlined before, using complex octonions and one new particle property: A mixing angle α models relative strength of gravitational compared to electromagnetic force.

Define:

$$\begin{aligned}\nabla_{\text{A}} &:= \; i_1\,(\;\;\partial_0 - i_0 e A_0)\,, \\ \nabla_{\text{B}} &:= \; i_5\,(\;\;\partial_3 - i_0 e A_3) \\ &\quad\; + i_6\,(-\partial_2 + i_0 e A_2) \\ &\quad\; + i_7\,(\;\;\partial_1 - i_0 e A_1)\,, \\ \nabla &:= \nabla_{\text{A}} + e^{i_0\alpha}\nabla_{\text{B}},\end{aligned} \tag{27}$$

and:

$$\begin{aligned}\Psi_{\text{A}} &:= \quad\;\; \psi_0 + i_1\psi_1 + i_2\psi_2 + i_3\psi_3, \\ \Psi_{\text{B}} &:= -i_4\psi_4 + i_5\psi_5 + i_6\psi_6 + i_7\psi_7, \\ \Psi &:= \Psi_{\text{A}} + e^{i_0\alpha}\Psi_{\text{B}},\end{aligned} \tag{28}$$

to obtain the unified equation of motion for a spin ½ particle:

$$(\nabla - m)\,\Psi \;=\; 0. \tag{29}$$

Choosing a mixing angle $\alpha=\pi/2$ reduces this formulation to the Dirac equation with electromagnetic field, suitable for all known elementary building blocks of matter. For other α values, the classical electromagnetic A_μ become a generalized four-potential, describing gravity as well. Similarly, the particle's electric charge e becomes a generalized charge.

All known particles have an α value that differs from $\pi/2$ only by the relative strength of gravity over electromagnetism — which is so small that one can't avoid the old, nagging question: Why even bother with quantum gravity, if measuring it directly seems to be a pipe dream? How could one formulation of quantum gravity ever be distinguished from another? How could one be falsified?

A new twist on isospin symmetries

We desire algebraic simplicity to describe physical law with fewer assumptions. But what are the deeper concepts behind physical forces on non-associative background? What does octonion rotation mean in nature? What quantities are preserved?

We note that final answers to these important questions are outstanding; but venture a speculation on how it could all fit together: A unified treatment of space-time and isospin symmetries on non-associative background may be suitable to unify known fundamental forces.

Non-associative decomposition

To support such a claim, that non-associative algebra can describe *all* fundamental forces of nature, it must be shown that quantum mechanics at its foundation can be expressed on a wider, non-associative background.

Such decomposition of the supersymmetric momentum operator and the angular momentum operator is possible, on split and complex octonion algebra respectively [8]. Known, proven laws may indeed have a hidden, non-associative structure from which they emerge.

Preserved quantities and symmetries

Then, one would want to ask for preserved quantities: If classical parameters, like space and time, only emerge from non-associative background, what are the invariant properties that warrant universal applicability of physical law in equivalent frames of reference? Two such invariants were pointed out earlier: The generalized length element $\|s\|^2$ (equation 21), and mass m (equation 29).

The symmetry that leaves the length element $\|s\|^2$ invariant when rotating the split-octonion basis elements, is by direct product of $\mathrm{O}(3,\ 4)$-boosts and real non-compact form of the exceptional group G_2 [3].

As for mass m on complex octonions, one might want to speculate that the exceptional Lie groups play a role as well in describing equivalent sets of basis elements; namely, the compact real form of E_6 (the “isometry group of the bioctonionic projective plane” [2]) comes to mind.

Fact and speculation

In the description of both electromagnetism and gravity outlined before, the respective fields are generated through simple one-parameter rotation on complex octonion algebra (equations 27 and 28). In the special case of electromagnetism, this becomes traditional $\mathrm{U}(1)$ isospin. But in the generalized case, the non-associative model makes no distinction between space and time geometry, and the symmetry that governs the respective force.

Whether this can be extended, to include the remaining fundamental forces, is pure speculation: Could the fabric of the physical universe be governed by a master geometry that does not distinguish space-time from isospin symmetries? The general direction appears worth exploring; octonionic spinors have already been shown to reproduce fermion generations of the Standard Model with the correct quantum numbers [9].

One subtle observation

At last, there is one more observation: Multiplying the split-octonion length element (21) with ε_4 leaves its norm $\|s\|^2$ invariant, but replaces space and time with their canonical transforms. Similarly, multiplying ∇ from (27) with i_0 only exchanges space-time derivatives with the corresponding field components. In both cases, it appears that an arbitrary choice of algebraic basis may replace parameters with the dynamic variables and fields they describe. Is this just coincidence, or does it hint toward an underlying symmetry that does not distinguish between observables and the very parameters used to model them?

Summary and outlook

In this essay, we have highlighted select findings, put them in context, and drafted a framework that could make all of it fit together. We have offered food for thought, and maybe reached deeper into speculative territory than some may find reasonable. There is much fascination, if irrational, about the possibility of describing all known fundamental forces through one master geometry. In the proposed interpretation, we have argued that it is human observation that creates the difference between parameters like space, time, and geometric magnitudes on one side, and physical dynamical properties like momentum, energy, and mass on the other.

Remaining are hard questions that need an answer: How does such an approach relate to classical quantum field theory? Why are the fundamental forces so very different in strength? It seems counterintuitive that gravity should be so much weaker than electromagnetism.

Human experience may also revolt against the proposition that *time* would merely be some watered-down aggregate, emergent from a background that defies intuition. Why would time appear so much different to us electromagnetic creatures? Why do we feel that there is only one reality (the "Now") which seemingly moves along the local observer time axis? What would be the generalized concept of things evolving?

Gravitating to the lighter side

Often it is hard, if not impossible, to distinguish a pioneer's vision from blissful illusion; which is why we end on a humorous note, and offer a healthy dose of irony:

The not-so-serious summary

"First, we rewrite quantum mechanics because high-school math (complex numbers, matrix algebra) leads to absurdities, such as observables and Bell's theorem. Misguided concepts to describe nature, e.g. 'space' or 'time', become a thing of the (redefined) past, and we'll rewrite relativity as well, while at it. After all, it was only Heisenberg's uncertainty that led regulators to limit the speed of light. We acknowledge that customary approximations, such as $E=mc^2$, have had sporadic successes in physics — namely, in describing everything that has ever been measured by mankind. We'll unify a couple of those isolated ideas (General Relativity, Standard Model), but – out of pure humbleness – separate our explanation of existence, consciousness, and life into a follow-up essay."

Many thanks and gratitude to John A. Shuster for inspiring and valuable discussions, comments, and corrections; and to Natalie S. Koeplinger for pointing out Kahlil Gibran's "Prophet", and all the love and patience with Jens. This essay was prepared using free software only, namely OpenOffice.org Impress, the Gimp, LyX, and LaTeX, on Linux. Thanks to the Wikipedia (en) user Althepal for releasing the prior picture of the digital clock to the public domain.

[1] **V. Dzhunushaliev,** *J. Math. Phys.,* **49**, 042108 (2008); arXiv:0712.1647;
Adv. High Energy Phys., **2007**, 12387 (2007); arXiv:0706.2398.
[2] **J. C. Baez,** *Bull. Amer. Math. Soc.,* **39**, 145 (2002); arXiv:math.ra/0105155.
[3] **M. Gogberashvili,** arXiv:0808.2496.
[4] **M. Gogberashvili,** *J. Phys. A*, **39** 7099 (2006); arXiv:hep-th/0512258.
[5] **T. Tolan, K. Özdaş and M. Tanışlı,** *Nuovo Cim.,* **121**, 43 (2006).
N. Candemir, M. Tanışlı, K. Özdaş and S. Demir, *Z. Naturforsch.,* **63a**, 15 (2008).
[6] e.g. **G. W. Gibbons and S. W. Hawking** (ed), *Euclidean Quantum Gravity* (World Scientific, Singapore, 2004).
[7] **J. Köplinger,** *Appl. Math. Comput.,* **188**, 954 and 948 (2007); preprints at http://jenskoeplinger.com/P.
[8] **V. Dzhunushaliev,** arXiv:0805.3221.
[9] **T. Dray and C. A. Manogue,** in *Clifford Algebras and their Applications in Mathematical Physics* (Birkhäuser, Boston, 2000); arXiv:hep-th/9910010.