Notes towards a nonassociative quantum theory

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Why nonassociative quantum theory?

Emergence: More observable effects can be described from a single underlying theory.

Reductionism: Such theory is simpler than the combined theories that describe each effect individually.

Fundamental forces as of 2010

Today's field theories (FT), quantum field th. (QFT), operator quantum th. (OQT), and unified th. (UT):

Force	FT	QFT	OQT	UT	Small print
Strong	✓	✓*	✗	✗*	(*) confinement problem, difficult to quantize, near-side ridge (RHIC, LHC), infinite nature constants, no GUT (simplicity gain when unified with electroweak force)
Weak	✓	✓**	✗	✓	(**) CP violation, neutrino masses
EM	✓	✓	✓	✓	
Grav	✓	✗***	✗	✗	(***) Planck scale, effects near singularity, superposition of states

In this talk

Talk with touch on:

1. How a nonassociative quantum theory (to be found) can model the hypothetical strong force "glueball" particle, and
2. a prototype for such theory in one dimension.

Part 1

Strong force and glueball

The strong force between quarks is difficult (quantization, confinement problem), and the algebra of quantum operators describing strongly interacting fields is unknown.

What could such an algebra describe? Suggestion here: The glueball, a hypothetical particle made from gluons only. [1,2]

Assume that the strong force field operators, $A_\mu^B(x^\mu)$, can be decomposed as:

$$A_\mu^B = e_\mu^{\bar{i}} \phi^{\bar{i}B}$$

(here, B is a label for one of the fields A^B, A^C, etc; μ enumerates space-time dimensions).

Field decomposition more formally:

$$\begin{aligned} e_\mu^{\bar{i}}, \phi^{\bar{i}B} &\in \mathbb{A}, \\ A_\mu^B := e_\mu^{\bar{i}} \phi^{\bar{i}B} &\in \mathbb{G}, \\ \mathbb{G} &\subset \mathbb{A}. \end{aligned}$$

$\mathbb{A}$ is an algebra with nonassociative product, and $\mathbb{G}$ is an associative subalgebra.

We can now propose: Observation requires reassociation

Propose modified Born rule: Observable effects from operators $\left\{A_\mu^B, A_\mu^C, \ldots\right\}$ are from their reassociated parts:

$$
\begin{aligned}
A_\mu^B A_\mu^C \left|\psi\right\rangle &= \left(\left(e_\mu^{\bar{i}} \phi^{\bar{i}B}\right)\left(e_\mu^{\bar{i}} \phi^{\bar{i}C}\right)\right)\left|\psi\right\rangle \\
&= \left(\left(\left(e_\mu^{\bar{i}} \phi^{\bar{i}B}\right) e_\mu^{\bar{i}}\right) \phi^{\bar{i}C}\right)\left|\psi\right\rangle \\
&\quad + \left(\tilde{m}_2\right)_{\mu\nu}^{BC}\left|\psi\right\rangle ,
\end{aligned}
$$

where $\left(\tilde{m}_2\right)_{\mu\nu}^{BC}$ is the associator.

Schematically, the modified rule for observability (obs) is:

Operator	obs
$\left(A_\mu^B\right)\|\psi\rangle = \left(e_\mu^{\bar{i}}\phi^{\bar{i}B}\right)\|\psi\rangle$	✗
$\left(A_\mu^B A_\mu^C\right)\|\psi\rangle = \left(\left(e_\mu^{\bar{i}}\phi^{\bar{i}B}\right)\left(e_\mu^{\bar{i}}\phi^{\bar{i}C}\right)\right)\|\psi\rangle$	✗
$\left(\left(\left(e_\mu^{\bar{i}}\phi^{\bar{i}B}\right)e_\mu^{\bar{i}}\right)\phi^{\bar{i}C}\right)\|\psi\rangle$	✓
$(\tilde{m}_2)_{\mu\nu}^{BC}\|\psi\rangle$	✓

Constrain to physics of a SU(3) gauge field

The physics of the strong force requires an SU(3) gauge field. After many prerequisites:

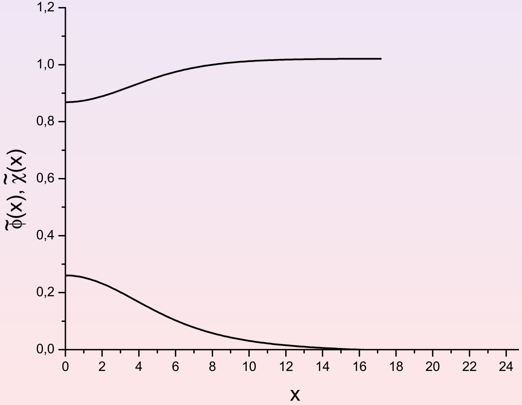
$$\mathcal{L}_{eff} = \langle F^{B\mu\nu} F^{B}_{\mu\nu} \rangle \approx \frac{1}{2} \left(\partial_\mu \chi \right)^2 + \frac{1}{2} \left(\partial_\mu \phi \right)^2 - V \left(\phi, \chi \right),$$

where the potential $V(\phi, \chi)$ is:

$$V(\phi, \chi) = \frac{\lambda_1}{4} \left(\chi^2 - m_1^2 \right)^2 + \frac{\lambda_2}{4} \left(\phi^2 - m_2^2 \right)^2 + \frac{\lambda_3}{2} \phi^2 \chi^2 + C.$$

Solutions exist for the glueball

The Lagrangian and potential describe a gluon field without quarks, i.e., a glueball. Regular solutions with finite energy exist.



Conclusion for part 1

A nonassociative decomposition of a SU(3) gauge field in physics can:

- construct an operator algebra of strongly interacting fields, and
- obtain a model of the glueball.

Part 2

Prototype in one dimension

What makes a new nonassociative quantum theory "simple"? Don't know until we have one.

Here: Build a prototype in one dimension that is built from a self-duality principle, works on $\mathbb{C}$, $\mathbb{H}$, and $\mathbb{O}$, and is "promising".

Assumption: Self-dual transformations

Define transformations T in the form of exponentials:

$$T \sim \alpha^{\beta}.$$

Require a self-duality principle when modeling a quantum system:

$$\alpha^{\beta} \sim \tilde{\beta}^{\tilde{\alpha}}.$$

Needs algebras that have exp and ln.

For vector space $\mathbb{R}^d$ with dimension d define:

$$T^A, T^P \ : \ \mathbb{R} \rightarrow S^{(d-1)},$$

$$\begin{aligned} \text{Active} \quad T^A(x) &:= |x-a|^{\theta t_A}, \\ \text{Passive} \quad T^P(x) &:= \theta^{(x-a)t_P}, \end{aligned}$$

where

$$a, t_A, t_P \in \mathbb{R}, \qquad x \neq a, \qquad S^{(d-1)} \subset \mathbb{R}^d,$$
$$\theta \in S^{(d-1)}, \ \ \theta^2 = -1, \ \ y \in S^{(d-1)} \Rightarrow |y| = 1.$$

Equivalence of active and passive transformations

Define functions f_*, g_*:

$$\begin{aligned} f_* \ &: \ T_1 \otimes \ldots \otimes T_n \rightarrow \left[S^{(d-1)}\right], \\ &T_i \in \left\{ T_i^A, T_i^P \right\} \text{ for } i = 1, \ldots, n, \\ g_* \ &: \ \tilde{T}_1 \otimes \ldots \otimes \tilde{T}_n \rightarrow \left[S^{(d-1)}\right], \\ &\tilde{T}_i \in \left\{ \tilde{T}_i^A, \tilde{T}_i^P \right\}. \end{aligned}$$

Then equivalence of active and passive transformations, $\alpha^\beta \sim \tilde{\beta}^{\tilde{\alpha}}$, can be proposed:

$$\forall f_* \exists g_* \ : \ f_* = g_* \text{ for } x \notin \{a_i\}.$$

At last, observation: By choice, associate T_A with fields and its dual T_P with particles. A differential operator $\hat{D}$ then models an observable quantity $m \in \mathbb{R}$ if:

$$\hat{D} f_* = m f_*,$$

and m invariant under $\{\mathbb{C}, \mathbb{H}, \mathbb{O}\}$ multiplication rule change for a fixed, orthogonal basis $\{1, \ldots, i_d\}$ of $\mathbb{R}^d$.

Nothing new on the complex numbers

On $\mathbb{C}$ to basis $\{1, \imath\}$, define operator

$$\hat{D} := -\imath\frac{\partial}{\partial x} - \sum_{i=1}^{n-1} \frac{t_i}{x - a_i},$$

$$\psi(x) := \imath^{t_n x} \prod_{i=1}^{n-1} |x - a_i|^{\imath t_i},$$

one particle $\imath^{t_n x}$ and $(n-1)$ fields:

$$\hat{D}\psi = m\psi,$$

$$m = \pi t_n \left(\frac{1}{2} \pm 2M\right), \qquad M \in \mathbb{N}.$$

Quaternions and octonions model local, multiparticle effects

When using quaternions $\mathbb{H}$:

- Local effects only (all a_i must be same),
- all $\theta_i \in \mathfrak{su}(2)$ algebra.

When using $\mathbb{O}$ to basis $\{1, \theta_i\}$:

- Local effects only, at least 3 particles,
- $\mathrm{aut}(\{\theta_i\}) \cong \mathrm{G}_2$ (or $\mathrm{SU}(3) \subset \mathrm{G}_2$), but ...
- ... $\theta_i \in \mathbb{O}$ nonassociative; need quasigroups?

Next steps

Physics: Add time and space dimensions.

Math: The f_* on $\mathbb{O}$ are (nonassociative) functions on the 7-sphere, $\left[S^7\right]$. The set of automorphisms on all possible $\mathbb{O}$ multiplications, that are keeping the vector space basis $\{1, \theta_i\}$ unchanged, forms the $\mathbb{Z}_2^4$ group [3,4,5]. Therefore, interest in Hopf coquasigroup $\left[S^7\right] \rtimes \mathbb{Z}_2^4$. [6,7]

Thank you for your attention!

References

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