Hidden nonassociative structures in quantum mechanics

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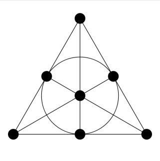
June 22nd, 2009



General context

Ultimate goal of this research:

- Build a complete, consistent nonassociative quantum theory.
- Investigate its consequences.
- Model more physical forces with fewer assumptions.

Overview (1)

This talk is divided into 2 parts:

1. Nonassociative quantum theory,
2. Possible applications.

Part 1: Nonassociative quantum theory

- *Unobservables* from nonassociative parts of an operator.
- Supersymmetric nonrelativistic Hamiltonian.
- Nonassociative decomposition of supersymmetric momentum operator, spin operator, and strongly interacting fields.

Overview (2)

Part 2: Possible applications

- Heisenberg uncertainty and light cone on split-octonions.
- Dirac equation with electromagnetic field on split-octonions.
- Four dimensional Euclidean operator quantum gravity on complex octonions.
- Generalized dimensional reduction on octooctonions.

If we run out of time, talk is online at:

http://jenskoeplinger.com/P/Denver09.pdf

Part 1:

Nonassociative quantum theory

Classical quantum mechanics

Classical quantum mechanics:

- Wave functions Ψ generally unobservable,
- operators H,
- decompose Ψ into orthogonal eigenfunctions ψ_n,
- eigenvalues h_n are observable.

Expectation values from classical quantum mechanics

$$\langle\Psi|\,H\,|\Psi\rangle = \sum_n h_n \int_V \psi_n^* \psi_n \, dV \qquad (1)$$

Nonassociative parts of an operator

Consequences from nonassociative parts of an operator H:

- Expression "$\left\langle \Psi \right| H \left| \Psi \right\rangle$" ambiguous, since:

$$\int_V \psi_n^* \left(h_n \psi_n \right) dV \neq \int_V \left(\psi_n^* h_n \right) \psi_n dV, \tag{2}$$

- loss of unitarity ("$\left\langle \Psi \right| HH^* \left| \Psi \right\rangle$" = ?) $\longrightarrow$ probability not conserved,
- nonassociative parts are *unobservable* in principle.

Unobservables

Unobservables are modeled by nonassociative parts of an operator, and cannot predict a measurement outcome. This is distinct from traditional "hidden variables", which could be measured in principle.

Challenges

Physics: How to approach $\langle\Psi|\nabla|\Psi\rangle$?

Algebra: How to handle differentials on a nonassociative background that may contain zero divisor and nilpotent spaces?

Nonassociative structures required

Hidden, nonassociative structures must exist across all fields of quantum mechanics [1-5]:

	physical observables	hidden constituents
supersymmetric quantum mech.	Hamiltonian *(change over time)*	...
operator QM	momentum operator *(location in space/time)*	...
operator QM	spin operator *(internal freedom)*	...
quantum field theory	field operators *(dynamic interaction)*	...

Supersymmetric quantum mechanics

The Hamiltonian H in nonrelativistic, N=1 supersymmetric quantum mechanics, can be written with split-octonions u_j as $H = \frac{1}{2}\left(Q+\bar{Q}\right)^2$ with unobservables [2,4]:

$$Q = \sum_{i=1}^{3} (-p_j + iV_{,j})\, u_j^* = \sum_{i=1}^{3} \mathscr{D}_j u_j^*, \quad (3)$$

$$\bar{Q} = \sum_{i=1}^{3} (p_j + iV_{,j})\, u_j = \sum_{i=1}^{3} \bar{\mathscr{D}}_j u_j. \quad (4)$$

Mathematical challenge (open question)

- How to evaluate differential operators $\mathscr{D}_j$ on split-octonion background u_j that contains nilpotent and zero divisor spaces?
- Time dependency $\langle\Psi|\frac{dL}{dt}|\Psi\rangle = \langle\Psi|\frac{\partial L}{\partial t} + \frac{i}{\hbar}[H, L]|\Psi\rangle$? [4]

Supersymmetric momentum operator

The simplest supersymmetric algebra is defined as follows [3]:

$$\begin{aligned}
[P^{\mu}, Q_a] &= \sigma^{\mu}_{a\dot{a}} \bar{Q}^{\dot{a}}, && (5)\\
\left[P^{\mu}, \bar{Q}^{\dot{a}}\right] &= -\sigma^{\mu\dot{a}a} Q_a, && (6)\\
[M^{\mu\nu}, Q_a] &= -i\left(\sigma^{\mu\nu}\right)_a^{\ \dot{b}} \bar{Q}_{\dot{b}}, && (7)\\
\left[M^{\mu\nu}, \bar{Q}^{\dot{a}}\right] &= -i\left(\sigma^{\mu\nu}\right)^{\dot{a}}_{\ b} Q^b, && (8)\\
\{Q_a, Q_{\dot{a}}\} &= 2\sigma^{\mu}_{a\dot{a}} P_{\mu}, && (9)\\
\{Q_a, Q_b\} &= \left\{\bar{Q}_{\dot{a}}, \bar{Q}_{\dot{b}}\right\} = 0. && (10)
\end{aligned}$$

Algebraic challenge (open question)

Is an algebraic decomposition of P_{μ} possible such that

$$P_{\mu} = \frac{1}{4}\sigma^{a\dot{a}}_{\mu}\{Q_a, Q_{\dot{a}}\}? \qquad (11)$$

Spin operator

A nonassociative decomposition of the spin operator [3]:

$$[R^\mu, R^\nu] = 2M^{\mu\nu}, \tag{12}$$
$$[M^{\mu\nu}, M^{\rho\sigma}] = \imath(\eta^{\nu\rho}M^{\mu\sigma} + \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\mu\rho}M^{\nu\sigma} - \eta^{\nu\sigma}M^{\mu\rho}) \tag{13}$$
$$(R^\mu, R^\nu, R^\rho) = (R^\mu R^\nu)R^\rho - R^\mu(R^\nu R^\rho) = 2\varepsilon^{\mu\nu\rho\sigma}R_\sigma. \tag{14}$$

Solutions

- 3-component [3]: $R^i = -\frac{1}{4}\varepsilon_{ijk}[q_{j+3}, q_{k+3}]$
  (split-octonions q_μ *and* $i, j, k \in \{1,2,3\}$*).*
- 4-component [10]: $R^0 = \frac{i_4}{2}(1+\imath)$ and $R^j = \frac{i_{(j+4)}}{2}(1-\imath)$
  (complex octonions to basis $\{1, i_1, \ldots, i_7\}$ *with coefficients* $\{1, \imath\}$*).*

Quantum field theory

We assume that operators of strongly interacting fields $\Phi_m(x^\mu)$ can be decomposed into nonassociative constituents [5]:

$$\Phi_m(x^\mu) = f^i_\alpha(x^\mu)\, b_{i\beta}(x^\mu). \qquad (15)$$

This would lead to additional terms for interacting fields:

$$\left(\left(\left(f^{i_1}_{\alpha_1} b_{i_1\beta_1}\right) f^{i_2}_{\alpha_2}\right) b_{i_2\beta_2}\right) = \left(f^{i_1}_{\alpha_1} b_{i_1\beta_1}\right)\left(f^{i_2}_{\alpha_2} b_{i_2\beta_2}\right) + \text{associator}. \qquad (16)$$

Physics challenge (open question)

- Does such decomposition exist? What is the algebra?
- The quantum corrections from the associator may be similar magnitude as the fields. What is their physical meaning?
- Nonassociative parts of operators are unobservable; does this allow for a nonperturbative description of (unobservable) quarks and the strong force?

Summary: nonassociative structures

Summary of hidden, nonassociative structure in quantum mechanics:

	physical observables	hidden constituents
supersymmetric quantum mech.	Hamiltonian $H = \frac{1}{2}\left(Q + \bar{Q}\right)$	nonassoc. operators: Q, $\bar{Q}$ unobservables
operator QM	momentum operator $P_\mu = \frac{1}{4}\sigma_\mu^{a\dot{a}}\left\{Q_a, Q_{\dot{a}}\right\}$	Q_a, $Q_{\dot{a}}$ algebraic generators of SUSY
operator QM	spin operator $\hat{s}^i = -\frac{1}{4}\varepsilon_{ijk}\left[q_{j+3}, q_{k+3}\right]$	q_{j+3}, q_{k+3} are split-octonions
quantum field theory	field operators $\Phi_m\left(x^\mu\right) = f^i_\alpha\left(x^\mu\right) b_{i\beta}\left(x^\mu\right)$	f^i_α, $b_{i\beta}$ elements of nonassoc. algebra

Part 2:

Possible applications

Light cone and Heisenberg uncertainty

The invariant length element on split-octonions $\{1, j_1, j_2, j_3, I, J_1, J_2, J_3\}$, with $j_n^2 = -1$ and $I^2 = J_n^2 = +1$, is [6,8]:

$$s = c\,(t + I\hbar\omega) + \sum_{n=1}^{3} (J_n x_n + j_n \hbar\lambda_n)\,, \tag{17}$$

$$\|s\|^2 = \left[c^2 t^2 - |\vec{x}|^2\right] - \hbar^2 \left[c^2\omega^2 - \left|\vec{\lambda}\right|^2\right]. \tag{18}$$

Space $\vec{x}$ and time t are paired with their canonical transforms ω [energy^{-1}] and $\vec{\lambda}$ [momentum^{-1}].

Physics implications

- Maximum speed of light c and Heisenberg uncertainty $\hbar \leq \left|\frac{dt}{d\omega}\right|$, $\hbar \leq \left|\frac{dx_n}{d\lambda_n}\right|$ on the same geometrical footing.
- Open question: Does this hint at a unification of space-time and energy-momentum?

Dirac equation with electromagnetic field

The Dirac equation with electromagnetic field A_μ was expressed on split-octonions [7]:

$$\hbar\left[c\frac{\partial}{\partial t}+J^i\frac{\partial}{\partial x^i}\right]\psi+\left[-\left(mc+\frac{e}{c}A_0\right)+\frac{e}{c}A_nJ^n\right](I\psi) = 0. \quad (19)$$

The wave function ψ is a split-octonion, too. The fields are associated differently to ψ as compared to the differentials:

$$\nabla\left|\Psi\right\rangle+\mathrm{A}\left|I\Psi\right\rangle=0. \quad (20)$$

Physics considerations (open questions)

- How to calculate general expectation values?
- Maybe "$\left\langle\Psi\right|f(\nabla,A)\left|\Psi\right\rangle$" notation is insufficient?
- What predictions come from the remaining algebraic freedoms?

Four dimensional Euclidean operator quantum gravity

Complex octonions (octonions $\{1, i_1, \ldots, i_7\}$ with complex coefficients $\{1, i_0\}$) allow to express the Dirac equation

$$(\nabla - m)\Psi = 0, \qquad \nabla = \sum_{n=0}^{3} \gamma^n E^n, \tag{21}$$

with electromagnetic field A_n (on Minkowski space-time) and a counterpart on four dimensional Euclidean background [9]:

$$\begin{aligned} E^n_{\mathrm{EM}} = E^n_{\mathrm{GR}} &= \partial_n + i_0 q A_n \qquad (n \in \{1,2,3\}), & (22)\\ E^0_{\mathrm{EM}} &= i_0 (\partial_0 + i_0 q A_0) \longleftrightarrow E^0_{\mathrm{GR}} = \partial_0 + i_0 q A_0, & (23)\\ \Longrightarrow E^0_{\mathrm{EM,GR}} &= \exp(\alpha i_0)(\partial_0 + i_0 q A_0). & (24) \end{aligned}$$

The γ^n are modeled as octonion basis elements.

Physics consideration (same open question)

Yet again: maybe "$\langle \Psi | f(\nabla, A) | \Psi \rangle$" notation is insufficient?

Octooctonions (octonions $\{1, i_1, \dots, i_7\}$ with octonion coefficients $\{1, i_{\dot{1}}, \dots, i_{\dot{7}}\}$) allow to generalize the Dirac equation further [10]:

$$(\nabla - m)\Psi = 0, \qquad \nabla = \sum_{n=0}^{3} \gamma^n E^n, \tag{25}$$

$$\gamma^n \longmapsto \{1, i_1, \dots, i_7\}, \qquad E^n \longmapsto G\left(\{1, i_{\dot{1}}, \dots, i_{\dot{7}}\}, \partial_n\right). \tag{26}$$

Chance at unification?

The algebra might be wide enough to include fermion generations [11], and the weak and strong force [10]:

Octonion index:	1	2	3	4	5	6	7
γ^n:	γ^1	γ^1	γ^2				γ^0
E^n (dotted index):				E^1	E^2	E^3	E^0
Symmetry:	$\mathrm{SU}(2)$						$\mathrm{SU}(1)$
	$\mathrm{SU}(3)$						$\mathrm{SU}(1)$

Conclusion

Nonassociative quantum theory is hot!

- Chance at nonperturbative color symmetry.
- Chance at fermion generations from algebraic generators.
- Chance at quantum gravity and unification.
- Chance at QFT *and* operator QM for all forces.

Next, do you want to work on ...

- ... generalizing prediction of expectation values?
- ... realizing color force nonperturbatively?
- ... supplying a weak and strong field on octooctonions?

Acknowledgments

Many thanks to the conference organizers and the NSF, who made Prof. Dzhunushaliev's participation possible through a travel grant.

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